NCAAPMT Calculus Challenge Problem #10

Due: March 11, 2008

For each of the functions below, use the information about the function to determine its equation.

1) Let *g* be a differentiable function , defined for all real numbers *x*, with the following properties:

(i) $g'(x) = ax^2 + bx$

(ii) $g'(1) = 6$ and $g''(1) = 18$

(iii) $\int_1^2 g(x)\,dx = 18$

Find $g(x)$. (2 pts)

2) Let *h* be a differentiable function , defined for all real numbers $x \geq 0$ such that $h(0) = 5$ and $h(3) = -1$. Suppose that for any number $b > 0$ the average value of $h(x)$ on the interval $0 \leq x \leq b$ is $\frac{h(0) + h(b)}{2}$.

a) Find $\int_0^3 h(x)\,dx$. (1 pt)

b) Prove that $h'(x) = \frac{h(x) - 5}{x}$. (1 pt)

c) Use parts a) and b) to find $h(x)$. (1 pt)