

Problem #2 Solutions are due by October 1

One of the fundamental theorems upon which calculus is based is known as the Intermediate Value Theorem.

The Intermediate Value Theorem: If $f(x)$ is a continuous function on the closed interval $[a, b]$, then for every value M between $f(a)$ and $f(b)$, there is at least one value $c \in [a, b]$ for which $f(c) = M$.

From the Intermediate Value Theorem we can conclude, for example, that if a continuous function has a value less than 0.8 at $x = 0$ and a value greater than 0.8 at $x = 1$, then the function must have a value of exactly 0.8 at least once in the interval $[0, 1]$. On a continuous function we can't go from below 0.8 to above 0.8 without passing through 0.8.

At this point in your calculus course, you may not have encountered this important theorem. If you haven't, look up the Intermediate Value Theorem in your text and discuss it with your teacher.

Other *intermediate value properties* are often of interest as you will see in this problem.

Early in the season, Pat was hitting fewer than 80% of her free throws. At the end of the season, she was hitting more than 80% of her free throws.

- a) Must there have been a time during the season at which she was hitting exactly 80% of her free throws? If so, explain why. If not, give a counter-example. (3 pts)
- b) If the answer to a) was yes, find all other values p which has this intermediate value property (you can't go from below p to above p without going through exactly p). If the answer to a) was no, are there any values of p for which this intermediate value property does hold? If not, explain why there can be no such values. (2 pts)