

NCAAPMT Calculus Challenge Problem #3

Solutions due on October 15

In class, you have learned the definition of derivative. We know that

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

whenever the limit exists.

The derivative of a function is another function, so we can find its derivative as well.

The derivative of the derivative is known as the second derivative and is denoted $f''(x)$.

- a) Find a limit definition for $f''(x)$ similar to $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ and using only values of f . (do not use f' in your limit statement) (3 pts)

$$f''(x) = \lim_{h \rightarrow 0} \frac{?}{?}$$

- b) Use this limit definition to show that if $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3$, then

$$f''(x) = 2a_2 + 6a_3x. \quad (1 \text{ pt})$$

- c) Generalize your limit definition to find the n^{th} derivative, $\frac{d^n f}{dx^n}(x)$, in terms of f only. (1 pt)