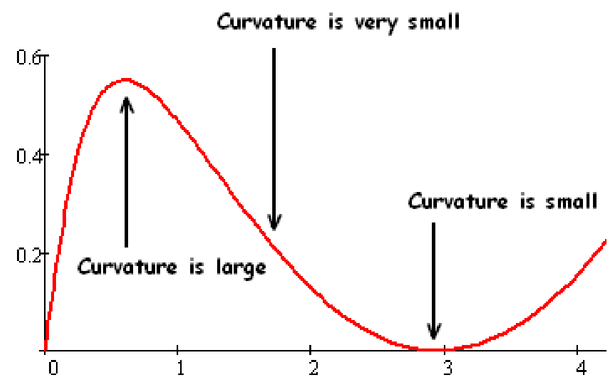
Problem #5 **Solutions Due on November 12**

The curvature of a function, denoted $\kappa(x)$, is the rate of change of the angle the tangent line makes with the horizontal with respect to distance traveled down the curve. If angle the tangent makes with the horizontal changes rapidly as you move with constant speed down the curve, then the curvature is large. If the angle changes slowly, then the curvature is small. This should match your intuition. The curvature is large when there is a "tight curve" in the graph.



In general, the curvature at a point describes how rapidly one has to turn in order to stay on the graph. If the curve is defined by the function $y = f(x)$, then the curvature can be computed using the equation

$$\kappa(x) = \frac{\left|f''(x)\right|}{\left(1+\left(f'(x)\right)^2\right)^{3/2}}.$$

By looking at the formula, it is easy to see that the curvature of a line is 0 at every point.

If a curve is given in parametric form, then $\kappa(t) = \dfrac{\left|x'(t)y''(t) - y'(t)x''(t)\right|}{\left(x'(t)^2 + y'(t)^2\right)^{3/2}}$.

a) Find the curvature of a circle of radius *R*. (2 pts)

b) Describe the relationship between the curvature of a function at a point and a circle tangent to the curve at that point. Such a circle is known as the *osculating circle* and we say that "traveling along a curve at a point where the curvature is *k* is like traveling along a circle whose radius is _____." (1 pt)

c) What is the maximum curvature of the quadratic $y = ax^2 + bx + c$ and where is this maximum curvature achieved? (1 pt)

d) At what point on the graph of $y = e^x$ is the curvature the greatest? (1 pt)