POW #6 **Due Tuesday, December 11**

This problem of the Bi-week is a mathematical modeling problem. If you like it, we can have more like this later in the year. I have my classes do at least one problem like this every 6 weeks. I usually give them a week to come up with a solution. Let me know what you think. Since we have the Thanksgiving break and we are starting two days late this week (my fault) the due date is December 11. If you have questions, just send me a note and I'll try to answer them.

Foraging For Food or "Do Bees Know Calculus?"

Foraging animals have an important problem to solve when their food supply is found in clusters or patches (acorns under trees, flowers growing in patches, worms in apples, etc). The problem is that, while food is easy to collect food when they first begin searching a new area, as the animal continues feeding, food becomes more and more difficult to find. Consequently, the foraging animal must search longer and longer for each additional morsel. At some point, the animal must decide to leave this patch and seek another, where food will be easier to find. However, it takes time to find and travel to the next patch, and the animal gains no food while it is traveling.

The choice about how long to stop for a feeding is one of the basic decisions for an organism that is searching for resources among widely scattered patches. Foraging animals have a classic max-min problem to solve. What is the optimal "giving up time" (when an organism should leave a patch that it is exploiting). When should the animal say enough is enough and move on to find the next patch? One crucial parameter that governs this decision is the travel time between patches.

For our example, we will consider a bee foraging for pollen on flowers that are dispersed across a yard. It gathers pollen rapidly when it first arrives at a new plant, but then finds it more difficult to pick up additional pollen. At some point, the bee must decide when to leave for "greener pastures". Figure 1 illustrates the consequences of this choice.

Suppose it takes 5 seconds to travel between flowers and the pollen collection function is

$$C(t) = \begin{cases} 0 \text{ if } t < 5 \\ 30 - 30e^{-0.2(t-5)} \text{ if } t \geq 5. \end{cases}$$

Figure 1 illustrates the difference in total pollen accumulation in a 40 second period if the bee forages for 5 seconds before moving on and foraging for 15 seconds before moving on.

Notice the total amount accumulated in a 40 second period is greater for 5 second foraging (about 76 μg) than for 15 second foraging (about 58 μg).

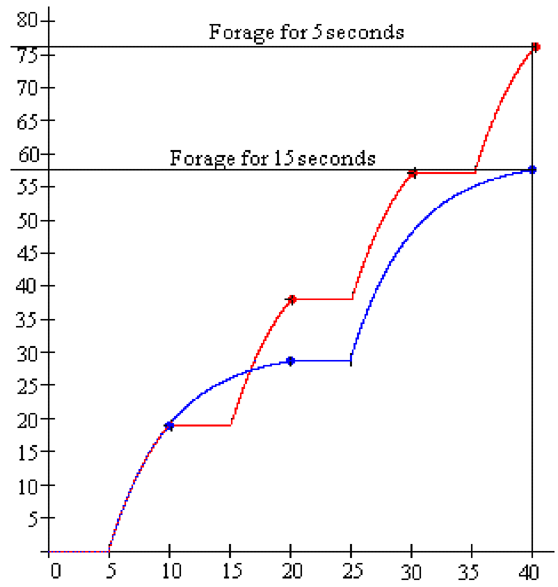


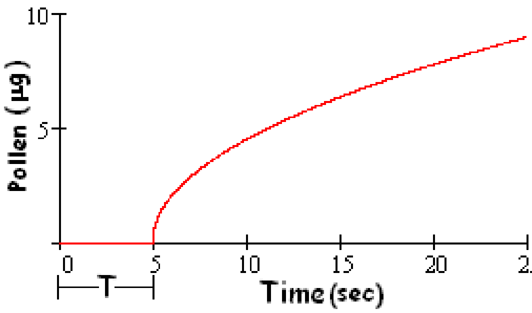
Figure 1: Comparing 5 and 15 second foraging times

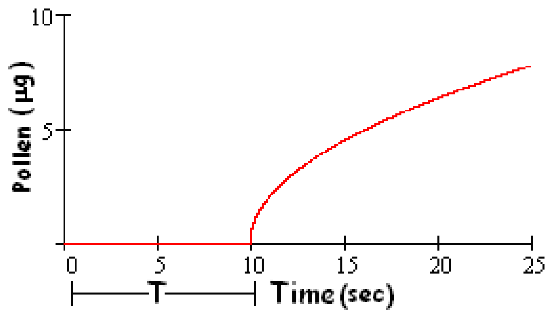
In this investigation, we want to determine the length of time that gives the largest total pollen accumulation in a fixed period of time.

The Effect of Travel Time and Rate of Collection

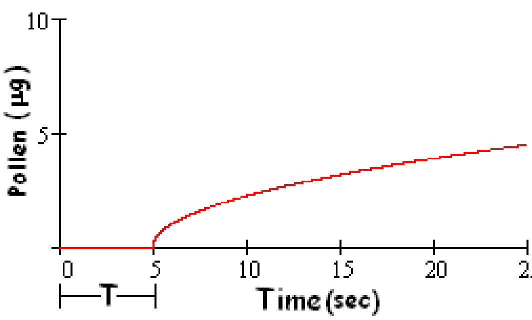
First, lets check our intuition. Upon what does the optimal foraging time depend? For each of the figures below, describe how the foraging time might depend on the time for travel between patches, T, and the shape of the function describing the energy gains over time.

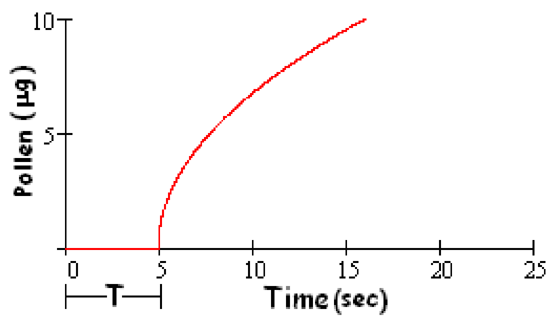
For the two collection curves shown below in which the travel times differ, would you expect the bee to spend longer or shorter periods of time on a flower if the travel time is longer?



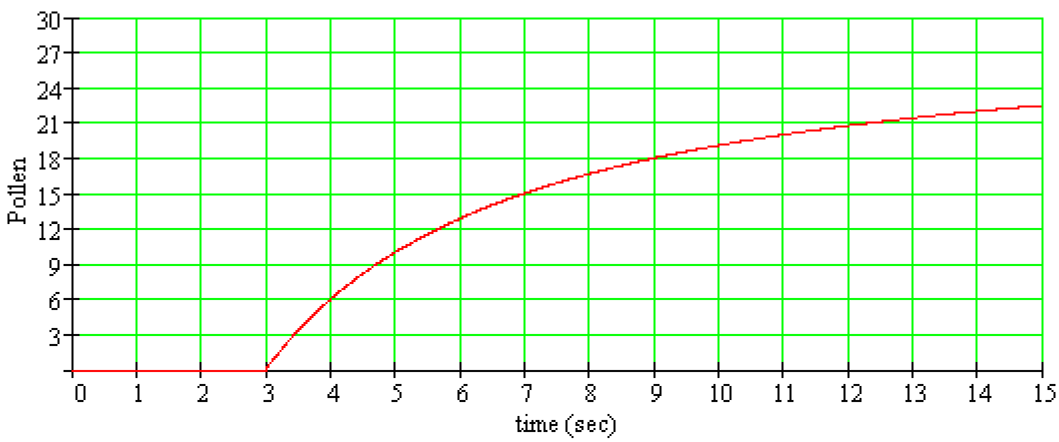


For the two collection curves shown below in which the travel times are the same, for which collection curve would you expect the bee to spend the longer time foraging?

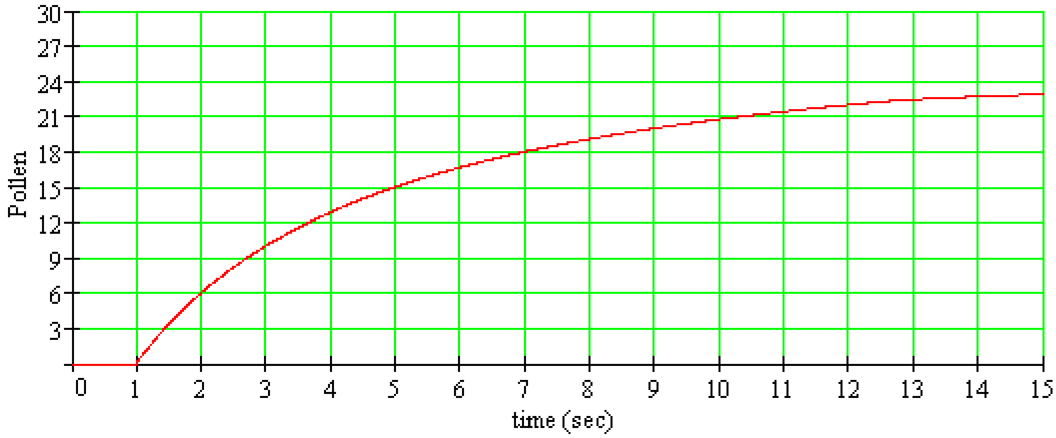




1) For the collection curve defined by the graph below, approximate the best foraging time. (1 pt)



2) If the travel time were reduced to 1 second, what happens to the location of the optimal foraging time? Does it increase or decrease? Was your intuition correct? (1 pt)



3) If the height of the curve is doubled, what happens to the location of the optimal foraging time? Does it increase or decrease. Was our intuition correct? (1 pt)

4) Consider the collection function $C(t)=\begin{cases} 0 \text{ if } t<T \\ A\sqrt{t-T} \text{ if } t\geq T \end{cases}$. What is the best foraging time in terms of *A* and *T*? Do the parameters *A* and *T* have the effect you expected? (1 pt)

5) For an arbitrary collection function $C(t)$, describe the location *t* at which the forager should leave its present situation in search of greener pastures? (1 pt)