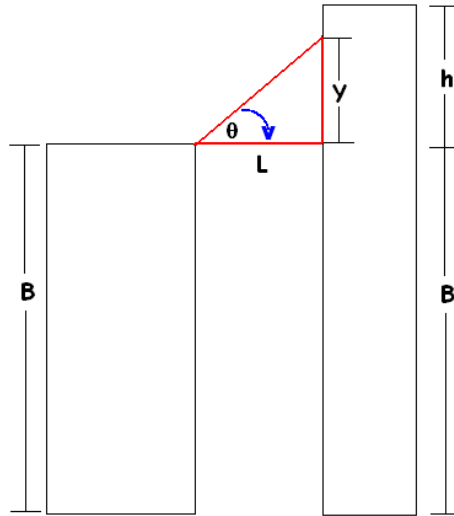


First, we draw a diagram. The figure at right represents the situation. We let the variable y represent the position of the brick measured from horizontal. So a negative value for y indicates a position below eye-level.

We see that $\tan(\theta) = \frac{y}{L}$, so $\theta = \text{Tan}^{-1}\left(\frac{y}{L}\right)$.

Also note that $\frac{d^2y}{dt^2} = -g$, so $\frac{dy}{dt} = -gt + c$. Since the initial velocity is 0 (the brick is dropped, not thrown), we have $\frac{dy}{dt} = -gt$.

Further, if $\frac{dy}{dt} = -gt$, then $y = h - \frac{1}{2}gt^2$.



If $\theta = \text{Tan}^{-1}\left(\frac{y}{L}\right)$, then $\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{y}{L}\right)^2} \left(\frac{y'}{L}\right)$. Simplify this expression and substitute

$y' = -gt$ and $y = h - \frac{1}{2}gt^2$. So, $\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{y}{L}\right)^2} \left(\frac{y'}{L}\right) = \left(\frac{L^2}{L^2 + y^2}\right) \left(\frac{y'}{L}\right) = \frac{Ly'}{L^2 + y^2}$.

Finally, $\frac{d\theta}{dt} = \frac{Ly'}{L^2 + y^2} = \frac{-Lgt}{L^2 + \left(h - \frac{1}{2}gt^2\right)^2}$.

We want to maximize $\frac{d\theta}{dt} = \frac{-Lgt}{L^2 + \left(h - \frac{1}{2}gt^2\right)^2}$.

Now we have a classic optimization problem. Differentiate using the quotient rule and set the derivative equal to zero.

$$\left(\frac{d\theta}{dt}\right)' = \frac{\left(L^2 + \left(h - \frac{1}{2}gt^2\right)^2\right)(-Lg) - (-Lgt)(2)\left(h - \frac{1}{2}gt^2\right)(-gt)}{\left(L^2 + \left(h - \frac{1}{2}gt^2\right)^2\right)^2}$$

The denominator is always positive, so we don't need to worry about a critical value where the derivative does not exist.

From the numerator, factor out $(-Lg)$ and multiply everything out, so

$$0 = (-Lg) \left(L^2 + h^2 - hgt^2 + \frac{g^2 t^4}{4} + 2ght^2 - g^2 t^4 \right).$$

Combine terms to arrive at the 4th degree equation: $0 = (L^2 + h^2) + hgt^2 - \frac{3g^2 t^4}{4}$.

We can solve this equation for t^2 , which is all we need to find $y = h - \frac{1}{2}gt^2$.

Using the quadratic formula, we have $t^2 = \frac{2h + 2\sqrt{4h^2 + 3L^2}}{3g}$. Since t^2 must be positive, we use the sum rather than the difference solution.

$$\text{So } y = h - \frac{1}{2}gt^2 = h - \left(\frac{1}{2}g\right) \left(\frac{2h + 2\sqrt{4h^2 + 3L^2}}{3g}\right) = \frac{2h}{3} - \frac{2\sqrt{4h^2 + 3L^2}}{3}.$$

Finally, the height of the brick at this point is $B + y = B + \frac{2h}{3} - \frac{\sqrt{4h^2 + 3L^2}}{3}$.

As a test case, consider $L = 50$, $g = 32$, $B = 150$, and $h = 30$. Then $t^2 = 2.82$, $y = -15.12$, and the height of the brick is 134.88 feet.

If the distance between buildings is 100 feet, then $L = 100$, $g = 32$, $B = 150$, and $h = 30$. Then $t^2 = 4.44$, $y = -41.10$, and the height of the brick is 108.9 feet.