Problem #8 **Solutions Due on January, 29**

A *n*th degree polynomial is a function of the form $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_nx^n$.

A curious way to write an *n*th degree polynomial is through the trigonometric definition

$$C_n(x) = \cos\left(n \cdot \text{Cos}^{-1}(x)\right),$$

where $C_n(x)$ is the *n*th degree polynomial.

If we rewrite this function using trig identities, we create an *n*th degree polynomial for each positive integer value of *n*.

a) Use trig identities to find the first five of these polynomials:

$$C_1(x),\ C_2(x),\ C_3(x),\ C_4(x), \text{ and } C_5(x). \quad (2 \text{ pts})$$

b) Determine the domain and range of these polynomials. Explain why the domains and ranges have the restrictions they do. (1 pt)

c) Find the value of $\int_{-1}^{1} C_n(x) \cdot C_{n+1}(x)\,dx$. (1 pt)

d) If you look at the graphs of the first 5 functions, all but one contain a particular point in common. What other polynomials $C_n(x)$ will miss this point. Can you explain why most of the graphs go through this point but some do not? (1 pt)