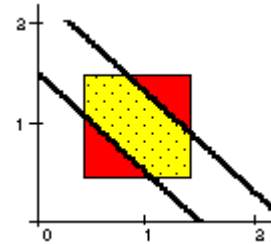


### Areas as Probabilities

Each problem below uses areas to determine probabilities of various occurrences. For example:

Given a pair of numbers that both round to 1, and assuming all pairs are equally likely, find the probability that their sum rounds to 2. For example, both 1.4 and 1.3 round to 1, but their sum 2.7 does not.

**Sample Solution:** Consider the two numbers  $x$  and  $y$ , with  $0.5 \leq x < 1.5$  and  $0.5 \leq y < 1.5$ . These are the numbers that round to 1. This gives us a sample space of a square. All points in the square represent possible values of  $x$  and  $y$ . If  $x + y \geq 1.5$  and  $x + y < 2.5$ , then the sum would round to 2. So, we want to find what fraction of the square  $0.5 \leq x < 1.5$  and  $0.5 \leq y < 1.5$  contains those points. We want to know the area inside the square between these two lines.



From the figure above, we see that  $P(1+1=1) = \frac{1}{8}$ ,  $P(1+1=2) = \frac{3}{4}$ , and  $P(1+1=3) = \frac{1}{8}$ .

The Problems below are similar in nature:

- 1 Given a pair of numbers that both round to 1, and assuming all pairs are equally likely
  - a) What is the probability their product rounds to 1? (1 pt)
  - b) What is the probability their quotient rounds to 1? (1 pt)
  
- 2) Suppose  $x$  and  $y$  are each chosen at random from  $(0, 1)$  with all values being equally likely. Compute the ratio  $R = \frac{y}{x}$ . What is the probability that the first non-zero digit in  $R$  (written as a decimal) is a 5? For example, 52.1772 and .053281 both have 5 as the first non-zero digit. (2 pt)
  
- 3) What is the probability that a dart, hitting a square board at random, lands nearer the center than the edge? (1 pts)