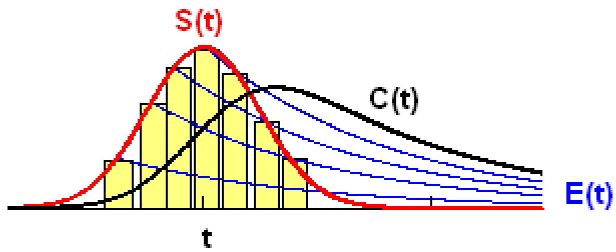
# Calculus Challenge #11

**Solutions Due March 25**

In modeling the concentration of hormones in the human body, simple models like the differential equation $\frac{dC}{dt} = -\alpha C(t) + S(t)$ are often used. In this model, $C(t)$ is the concentration of the hormone at time $t$ (which is being metabolized at a rate, $\alpha > 0$, proportional to the concentration at that time), while $S(t)$ is the rate at which the hormone is being secreted into the system.

A more sophisticated model for the concentration is

$$C(t) = \int_{-\infty}^{t} S(x)E(t-x)\,dx,$$




where $S$ is the rate of secretion and $E$ the rate of elimination. For most hormone secretions, $S(t)$ is either a pulse function or a variant of a Gaussian function (shown above).

1) Use technology to sketch a graph of the concentration function $C(t) = \int_{-\infty}^{t} e^{-2x^2} e^{-0.6(t-x)}\,dx$.

2) For the integral model, $C(t) = \int_{-\infty}^{t} S(x)E(t-x)\,dx$, find $C(t)$ if $E(t) = e^{-\alpha \cdot t}$ and $S(t) = S$ (constant). Does $\frac{dC}{dt} = -\alpha C(t) + S(t)$ for this model?

3) If $C(t) = \int_{-\infty}^{t} S(x)e^{-\alpha(t-x)}\,dx$, with $S(t)$ a non-constant secretion rate, we want to find $\frac{dC}{dt}$. To accomplish this, we need to use the definition of derivative, l'Hopital's rule, and the 2nd Fundamental Theorem of Calculus.

a) First, find $C(t+h)$.

b) Show that $C(t+h) - C(t) = \left(e^{-\alpha h} - 1\right)C(t) + e^{-\alpha h}\int_{t}^{t+h} S(x)e^{-\alpha(t-x)}dx$.

c) Find $\lim_{h \to 0} \frac{C(t+h) - C(t)}{h}$.