Calculus Challenge #12 **Due, Wednesday, April 8**

One afternoon, I filled a 25-gallon cooler half-full with water, put a yardstick in it, and opened the stop-cock to let the water out. I took measurements periodically as the cooler emptied. The data I gathered is in Table 1 below. The cooler was cylindrical in shape, so $V = \pi r^2 h$, with a radius of 10 inches.

Time (secs)	0	30	60	100	140	180	210	265	330	390
Depth (in)	12.5	11.25	10.5	9.25	8.0	7.0	6.0	4.75	3.5	2.5

Table 1: Time and Depth of water in an urn

Under these conditions, the change in volume (*V*) is proportional to the square root of the depth (*h*) of the water, so, $\frac{dV}{dt} = -k\sqrt{h}$.

a) Solve the differential equation to find *h* as a function of *t*. What type of function is $h(t)$?

b) Fit a model of this type to the data in Table 1 and use that model to approximate *k* and the constant of integration. Present a graph of your model against the data.

c) After how many seconds is the depth *decreasing* at a rate of 0.025 inches/sec?

d) Suppose after 2 minutes water is added at $C\,\frac{\text{in}^3}{\text{min}}$. What value of *C* would keep the depth of the water constant?

Super Challenge: (0 points, just for fun, no calculus necessary)
e) When the cooler is full, the center of mass of the cooler with water is about in the center of the cooler (a little lower than the center since the top is not on the cooler). As the water flows out, the center of mass of the cooler-with-water drops, decreasing continuously. But, when the tank is empty, the center of mass is magically again in about the center of the cooler. So, at some point, while the water level is dropping, the center of mass began to rise. Describe the conditions at which the center of mass reaches its lowest level.