

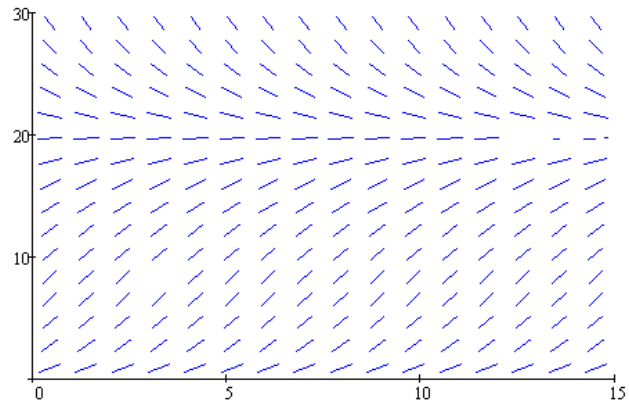
Calculus Challenge #8

Solutions Due February, 11

Observations on the growth of animal tumors indicate the size y of the tumor obeys the differential equation $\frac{dy}{dt} = -ky \ln\left(\frac{y}{b}\right)$ with k and b positive constants. This is known as the Gompertz growth law.

It is always a good first step to look at the slope field. At right we see the slope field for the differential equation using specific choices of k and b . All choices for k and b produce similar graphs.

This graph should help guide your investigation.



Slope field for $dy/dt = -k y \ln(y/b)$

- In terms of k and b , when is the tumor growing most rapidly?
- Solve the differential equation with the initial condition $y(0) = y_0$. If you have studied the technique known as “separation of variables”, you know how to do this. If you haven’t yet seen this technique, don’t worry, just solve the integral equation
$$\int \frac{dy}{y \ln\left(\frac{y}{b}\right)} = \int -k dt$$
 for y .
- Find a relation between the initial condition y_0 , and the constants k and b , so the graph of y vs t has no point of inflection.
- Find $\lim_{t \rightarrow \infty} y(t)$. What does this say about the growth of the tumor?
- Given the slope field graphed above, what values were used for k and b ?