Calculus Challenge #2

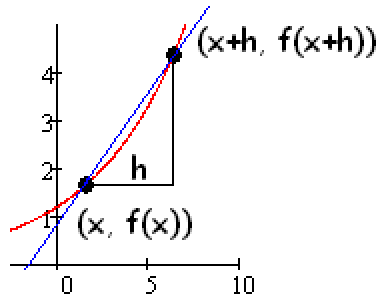
Solutions due October 15, 2008

This bi-week's problem deals with the definition of a derivative. We offer an alternative definition and ask how well it works. This is one of the more theoretical problems in the set. The next problem will be more computational in nature and deal with the product rule.

Classic definition of Derivative:

It is traditional to define the derivative as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

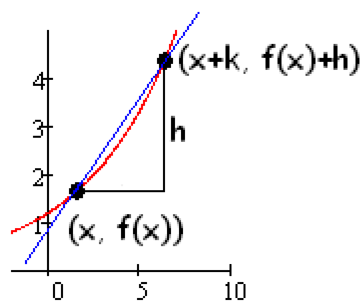
The secant line geometry of this suggests that we pick a horizontal distance *h*, take whatever change in *y* we need to create the secant line and compute the slope. We then shrink *h* to zero and consider the limiting value of the slope.



The "Modified" Derivative:

Why not do it the other way around? Why not fix a vertical distance *h*, take whatever change in *x* we need to create a secant line and compute the slope. Then shrink *h* to zero and consider the limiting value as before.

What happens if you do it this way?



a) Show that the value of *k* in the diagram above is $k = f^{-1}\left(f(x)+h\right)-x$.

b) This gives a new definition of derivative. Instead of $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ being represented by $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, we can also represent $\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$ by $\lim_{h\to 0}\frac{h}{f^{-1}\left(f(x)+h\right)-x}$. Use this definition to find the derivative of

i) $f(x)=\sqrt{x}$

ii) $f(x)=\frac{1}{x}$

iii) What difficulties arise with $f(x)=x^2$? Does the new definition work for all *x*?