

## Comparing Paper Towels

We are given the task of designing an experiment to compare the performance of several brands of paper towels. The materials available for the experiment are:

- Two rolls each of Brawny, Bounty, Viva, Scott, So-Dri, 7<sup>th</sup> Generation, and Sparkle brand paper towels.
- Stop watches
- Tape measures
- Measuring cups
- Embroidery hoops
- Eye droppers
- Pennies and 50 gram weights

First, consider what aspect of paper towel performance you wish to investigate. Design an experiment to compare the performance of 3 or more brands. Then conduct the experiment using at least 4 replications and analyze the data.

What are the questions we can ask about the paper towels? What do we want to measure and what kind of experimental design should be used? During the NCSSM Statistics Leadership Institute, participants designed experiments to compare towels on both strength and absorption. Some participant groups bought additional brands of paper towels or added the paper towels used in the school bathrooms. The following experiments were proposed by Institute participants.

### **Tests of towel strength:**

Strength was generally tested by the amount of weight a wet towel could hold before tearing. Some variations are seen in the examples below.

(1) Place a paper towel in the embroidery hoop to maintain a constant tension. Add one tablespoon of water to the center and wait 30 seconds. Then place a 50 gram weight in the center of the hoop, wait 3 seconds, and place another on top, continuing until the towel breaks. Record the weight on the paper towel before the towel broke. Larger weights correspond to stronger towels.

This experiment can be modified by changing the amount of water used and using pennies instead of 50 gram weights.

(2) Place a paper towel in the embroidery hoop. Place weights in a circle around the center of the towel. Place drops of water in the center of the towel. Count the number of drops until the towel breaks. In this experiment, the stronger towels will hold the weights with the larger amount of water.

(3) Attach a string to an embroidery hoop. Place a paper towel in the embroidery hoop. Then place 25 pennies on the towel in the hoop. Wet a rough surface, such as concrete, and drag the hoop across the wet surface. Measure the distance traveled before

the paper shreds and some pennies fall out. The greater the distance traveled, the stronger the towel.

(4) Place a paper towel in the embroidery hoop. Place 15 drops of water in the center. Wait 30 seconds. Measure the diameter of the water on the towel. Then stack pennies in the center of the hoop. Gently place a penny, wait 3 seconds, and continue to stack pennies until the towel breaks. Record the number of pennies before the towel broke. The measurement for comparison is the ratio of number of pennies to the square of the diameter of the spread of water  $M = \frac{\# \text{ pennies}}{\text{diameter}^2}$ . The larger the value of  $M$ , the stronger the paper towel.

### Tests of absorbency:

Absorbency was tested in one of two ways, speed of absorption and volume of absorption. When testing speed, the more rapidly the water moves through the towel, the more absorbent the towel. When testing volume, the more water a towel will hold, the more absorbent the towel.

(5) Place a paper towel in the embroidery hoop. Draw a circle 2 inches in diameter in the center of the towel.

a) Place drops of water in the center of the towel. Count the number of drops added until the water spreads out to fill the circle. The towels requiring fewer drops are the more absorbent towels.

b) Place 10 drops of water in the center of the circle. Measure the time it takes for water to spread out and fill the circle. The towels with shorter times are the more absorbent towels.

(6) Cut towels into strips 1 inch wide and 6 inches long. Draw marks 1 inch from each end with an overhead pen. Fill a cup with water. Hold each strip vertically with tweezers with the bottom end one inch in the water. Measure the time it takes for the water to travel from bottom mark (at the top of the water) to the top mark. This measures how quickly the towel absorbs water. The faster the water moves through the towel, the more absorbent the towel.

(7) Put one cup of water in a measuring cup. Submerge a towel in the water for 5 seconds. Remove the towel. Measure the amount of water remaining in the cup to determine the amount of water removed from the measuring cup. The greater the amount of water removed from the cup, the more absorbent the towel.

The size of the paper towels varies as does the cost. This experiment can be modified to compare

- a) a single sheet, since that's what people use
- b) a fixed area, either by a fixed area of towel or by using a whole towel and making measurement on a per unit area basis
- c) fixed price, either by using a section that costs 0.5 cents or by making the measurement on a fixed cost basis.

(8) Put one cup of water in a measuring cup. Submerge a towel in the water for 5 seconds. Remove the towel. Let the towel drip for 30 seconds back into the measuring cup. Measure the amount of water removed from the cup to determine the amount of water retained in the towel. The greater the amount of water retained in the towel, the more absorbent the towel. This experiment can be modified as in Experiment (7).

### **Design of Experiments**

All of these suggested experiments can be carried out using either a completely randomized design or a randomized complete block design. If the experimenters believe that they will become more skilled (or less skilled) at performing the experiment and making the measures as the experiment progresses, they should use a randomized complete block design, using each brand of towel in random order for each block. If no learning effect is expected, a completely randomized design should be used. Many of the following experiments designed by the participants utilize a randomized complete block design, using the trial number as the blocking variable. In the analysis it is clear that this blocking variable adds little to the experiment. A completely randomized design would give more power in most of these situations.

Note: Studies of a single factor are generally one of two types. In the first, a random sample is drawn from the population of interest and the levels of the factor are randomly assigned to these experimental units. In the second, random samples are taken from each of  $k$  levels of the factor. The paper towel studies are examples of this second type. It is not possible to randomly assign a paper towel to be “Bounty” or any other brand of paper towels. Instead, we took a random sample from each type of paper towel. Some textbooks refer to only the first type of study in which treatments are randomly assigned to experimental units as having a completely randomized design. Here, we use the term for both types as in Inman (1994, p. 691). However, the types of inference that can be drawn from each differs (see Figure and related discussion on page 12).

## Results of Participant Experiments

### Experiment # 1a

*Experimental Protocol:* Place a paper towel in the embroidery hoop to maintain a constant tension. Add one tablespoon of water to the center and wait 30 seconds. Then gently place a 50 gram weight in the center of the hoop. Wait 3 seconds and place another weight on top until the towel breaks. Record the weight on the paper towel before the towel broke.

This experiment was conducted as a completely randomized design using 5 brands of towel. The treatment structure has 1 factor (Brand) at 5 levels. The results are given in the table below:

Scott	So-Dri	Bounty	Brawny	Sparkle
800	450	950	450	300
700	500	1050	500	300
750	450	1000	450	300
700	450	1050	450	300

Weight By Brand

Oneway Anova

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	1267000.0	316750	271.5000
Error	15	17500.0	1167	Prob>F
Total	19	1284500.0	67605	<.0001

Means for Oneway Anova

Brand	Mean	Std Error
Scott	737.50	17.078
So-Dri	462.50	
Bounty	1012.50	
Brawny	462.50	
Sparkle	300.00	

Bounty held the most weight while Sparkle held the least. From the computer output, we see that there is a significant difference in the mean weight the towels will hold when wet. Using Fisher's *LSD*, we find that any difference larger than

$2.131\sqrt{\frac{2(1167)}{4}} = 51.476$  is considered significant. Bounty has a significantly larger mean weight before breaking than all of the others. Other significant differences are also present.

### Experiment #1b

*Experimental Protocol:* Place a paper towel in the embroidery hoop to maintain a constant tension. Add 10 drops of water to the center and wait 30 seconds. Then we place pennies in the center of the hoop until the towel breaks. Count the number of pennies before the towel broke. Repeat the procedure with 20 drops of water and with 30 drops of water.

This experiment was done as a randomized complete block with two repetitions with 4 brands of towel. The data are in the table below:

		School	Eco-Friendly	So-Dri	Brawny
Trial 1	10 Drops	266	87	220	260
	20 Drops	500	80	160	206
	30 Drops	400	135	230	270
Trial 2	10 Drops	479	90	174	215
	20 Drops	200	97	160	220
	30 Drops	378	122	160	200

The treatment structure has 2 factors; brand at 4 levels and amount of water at 3 levels. All 12 treatment combinations were used in random order once and then the process repeated with a new randomization. This is a 3 x 4 factorial arrangement of treatments in a randomized complete block design. The computer output of the analysis is given below.

Response: Number of Pennies

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	12	238221.50	19851.8	3.0761	0.0363
Brand	3	227856.12	75953.0	11.7691	0.0009
Water	2	1132.33	566.17	0.0877	0.9166
Brand x Water	6	4993.00	832.17	0.1289	0.9899
Trial	1	4240.04	4240.04	0.6570	0.4348
Error	11	70988.46	6453.5		
Total	23	309209.96			

Brand		Water		Trial	
Level	Mean	Level	Mean	Level	Mean
School	370.500	10 Drops	223.625	1	234.500
Eco-Fri	101.833	20 Drops	211.625	2	207.917
So-Dri	184.000	30 Droop	227.375		
Brawny	228.500				

From the output, we see that there is a significant difference in brands, but there was not a significant effect of the amount of water, and no significant brand by water interaction. The lack of a water effect could be anticipated from the data, since a larger number of pennies was often held when more water was on the towel. This suggests that the effect of adding more water is overshadowed by the great variation in the strengths of towels for each brand.

Also, we see that there was no effect of the blocking variable, so a completely randomized design would work just as well. Fisher's *LSD* for the Brand effect is

$$2.110 \sqrt{\frac{2(6453.5)}{6}} = 97.9$$

Based on the results of this experiment, we can say that the school towel has the largest mean number of pennies held and is significantly different from all other towels. The Brawny towel has a larger mean number of pennies held than Eco-Friendly, but is not significantly different from So-Dri. So-Dri and Eco-Friendly are not significantly different with respect to the mean number of pennies held.

**Experiment #2** None of the participant groups chose to conduct this experiment.

**Experiment #3**

*Experimental Protocol:* Attach a string to an embroidery hoop. Place a paper towel in the embroidery hoop to maintain a constant tension. Add 25 pennies to hoop. Wet surface of concrete and drag hoop across wet surface. Measure distance traveled (in hoop-lengths) before paper shreds and pennies fall through.

This experiment was run as a randomized complete block, using each of 6 brands once before repeating the experiment in a new random order, and using three replications. The treatment structure has 1 factor at 6 levels.

	Bounty	Brawny	School	Scott	So-Dri	Savon
Trial 1	10	3	4	5	0	0
Trial 2	10	3	3	8	0	0
Trial 3	12	4	3	8	0	0

Response: Distance

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	7	261.05556	37.2937	47.2736	<.0001
Brand	5	258.94444	51.788	65.6479	<.0001
Trial	2	2.11111	1.0555	1.3380	0.3055
Error	10	7.88889	0.7889		
Total	17	268.94444			

Brand Level	Mean	Trial Level	Mean
Bounty	10.6667	1	3.66667
Brawny	3.3333	2	4.00000
School	3.3333	3	4.50000
Scott	7.0000		
So-Dri	0.0000		
Savon	0.0000		

In this experiment, there is a significant difference in the mean distance the towel can be pulled before the pennies break through. There is no significant effect for the trials. A completely randomized design could be used if this experiment were to be

repeated. Fisher's *LSD* is  $2.228\sqrt{\frac{2(0.7889)}{3}} = 1.62$ . Clearly, the So-Dri and Savon

towels performed most poorly. The Brawny and school towels have a significantly larger mean distance than either of these two brands. The Scott towels have a significantly larger mean distance than do Brawny, the school towel, So-Dri, and Savon. Bounty is significantly larger than all of the others. Bounty performed best on this test.

**Experiment #4**

*Experimental Protocol:* Place a paper towel in the embroidery hoop to maintain a constant tension. Place 15 drops of water in the center. Wait 30 seconds. Measure the "length" and "width" of the spread of the water on the towel. Then stack pennies in the center of the hoop. Gently place a penny, wait 3 seconds, stack another on top until the towel breaks. Count the number of pennies before the towel broke. The measurement

for comparison is the ratio of number of pennies to the “area” of the spread of water

$$M = \frac{\text{\# pennies}}{\text{length} \cdot \text{width}}$$

This experiment with one factor (Brand) at 4 levels was conducted as a completely randomized design. The results are grouped in the table below for convenience.

Brawny	Bounty	7 <sup>th</sup> Generation	Sparkle
4	9.81	2.21	3.25
3.44	9.41	2.74	3.13
3.31	8.89	2.71	2.04

Ratio By Brand

Oneway Anova

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	3	93.568567	31.1895	141.7222
Error	8	1.760600	0.2201	Prob>F
Total	11	95.329167	8.6663	<.0001

Level	Mean	Std Error
Brawny	3.58333	0.27085
Bounty	9.37000	
7 <sup>th</sup> Gen	2.55333	
Sparkle	2.80667	

From the computer output we see that there is a significant difference in mean value of  $M$  among brands. Bounty has far and away the largest value, either due to a large number of pennies held or a small spread of water, or both. The value of Fisher’s

$LSD$  is  $2.306\sqrt{\frac{2(0.2201)}{3}} = 0.8833$ . Bounty has a significantly larger mean value of  $M$

than the other towels tested. Brawny is significantly stronger than 7<sup>th</sup> Generation, but Sparkle is not significantly different from either. One way to describe these relationships is to code towels whose mean values are not significantly different with the same letter and those whose mean values are significantly different with different letters. Here, we have

Paper Towel	Code
7 <sup>th</sup> Generation	a
Sparkle	a b
Brawny	b
Bounty	c

### Experiment #5b

*Experimental Protocol:* Place a paper towel in the embroidery hoop to maintain a constant tension. Draw a circle 2 inches in diameter in the center of the towel. Place 10 drops of water in the center of the circle. Measure the time in seconds it takes for water to spread out and fill the circle.

This experiment was run as a randomized complete block design. One sheet of each of the 4 towels (Scott, 7<sup>th</sup> Generation, Sparkle, and Brawny) was used in random order before repeating the process. The data and analysis are shown below.

	Scott	7 <sup>th</sup> Generation	Sparkle	Brawny
Trial 1	12.05	6.99	48.82	3.77
Trial 2	2.69	4.62	78.62	13.79
Trial 3	10.91	13.07	58.17	3.23
Trial 4	15.29	6.33	44.04	4.34

Response: Time (secs)

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	6	7474.1941	1245.70	14.7219	0.0003
Brand	3	7330.0550	2443.35	28.8760	<.0001
Trial	3	144.1391	48.046	0.5678	0.6500
Error	9	761.5370	84.62		
Total	15	8235.7311			

Brand Level	Mean	Trial Level	Mean
Scott	10.2350	1	17.9075
7 <sup>th</sup> Gen	7.7525	2	24.9300
Sparkle	57.4125	3	21.3450
Brawny	6.2825	4	17.5000

Again we see a significant difference in the mean time to fill the circle. There is no effect from the trial, so a completely randomized design could be used if this experiment were to be repeated. Fisher’s *LSD* is  $2.262\sqrt{\frac{2(84.62)}{4}} = 14.7$ . The three towels Scott, 7<sup>th</sup> Generation, and Brawny are indistinguishable, but the mean time to fill the circle for Sparkle is significantly greater than the others.

**Experiment #6**

*Experimental Protocol:* Cut towels into strips 1 inch wide and 6 inches long. Draw marks 1 inch from each end with an overhead pen. Fill a cup with water. Hold a strip vertical with tweezers and hold bottom end one inch in the water. Measure the time it takes for water to travel from bottom mark (at the top of the water) to the top mark. This measures how quickly the towel absorbs water.

This experiment was conducted using a completely randomized design. The treatment Brand has 8 levels with 3 replications of each.

School	Brawny	Viva	Scott	Bounty	7 <sup>th</sup> Gen	So-Dri	Sparkle
0.4	1.5	1.1	0.9	1.4	1.3	1.1	1.6
0.4	1.5	1.2	1.0	1.4	1.5	1.2	1.6
0.4	1.7	1.2	0.9	1.4	1.3	1.2	1.6

Time By Brand

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	7	3.2266667	0.460952	100.5714
Error	16	0.0733333	0.004583	Prob>F
Total	23	3.3000000	0.143478	<.0001

Means for Oneway Anova

Level	Mean	Std Error
School	0.40000	0.03909
Brawny	1.56667	
Viva	1.16667	
Scott	0.93333	
Bounty	1.40000	
7 <sup>th</sup> Gen	1.36667	
So-Dri	1.16667	
Sparkle	1.60000	

In comparing mean times for the water to travel up the strip, we find a significant difference in the brands. For this experiment, the value of Fishers *LSD* is

$$2.120 \sqrt{\frac{2(0.004583)}{3}} = 0.1172.$$

Mean times that differ by more than 0.12 are considered significantly different. Sparkle and Brawny have the slowest mean times (largest) and are significantly slower than all other brands. The school towels have the quickest mean time (smallest) and are significantly faster than all other towels.

Paper Towel	Code
School Towel	a
Scott	b
So-Dri	c
Viva	c
7 <sup>th</sup> Generation	d
Bounty	d
Brawny	e
Sparkle	e

**Experiment #7b**

*Experimental Protocol:* Put one cup of water in a measuring cup. Submerge a 20 cm by 20 cm section of towel in the water for 5 seconds. Remove towel. Measure the amount of water remaining in the cup to determine the amount of water removed from the measuring cup. Record the amount of water removed from the measuring cup.

This experiment was done as a randomized complete block with three repetitions. Each paper towel was used once in random order within a block, then the process repeated two more times. The data measured in ounces and analysis follow:

	Bounty	7 <sup>th</sup> Gen	Sparkle	Scott	Brawny	Viva	So-Dri	School
Trial 1	11	5.5	7.5	7	9	16	6	5
Trial 2	13	6	5	8	5	12.5	6	3
Trial 3	14	5	5	8.5	9	16	4	5

Response: Water

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	9	306.02083	34.0023	16.4267	<.0001
Brand	7	300.33333	42.9048	20.7275	<.0001
Trial	2	5.68750	2.84375	1.3738	0.2852
Error	14	28.97917	2.06994		

Total 23 335.00000

Brand	Mean	Trial Level	Mean
Bounty	12.6667	1	8.37500
7 <sup>th</sup> Gen	5.5000	2	7.31250
Sparkle	5.8333	3	8.31250
Scott	7.8333		
Brawny	7.6667		
Viva	14.8333		
So-Dri	5.3333		
School	4.3333		

In this experiment, we see a significant difference in brand, but no significant effect of the blocking variable. If this experiment were to be repeated, a completely randomized design could be used. Fisher's *LSD* is  $2.145\sqrt{\frac{2(2.06994)}{3}} = 2.52$ . Using this test, Bounty and Viva had significantly higher mean volume of water absorbed than the other towels. Viva and Bounty are not significantly different. Brawny and Scott had significantly higher mean volumes absorbed than the school towels.

**Experiment #7b**

*Experimental Protocol:* Put one cup of water in a measuring cup. Submerge a towel in the water for 5 seconds. Remove the towel. Measure the amount of water remaining in the cup to determine the amount of water removed from the measuring cup. Measure the area of the towel used and compute the amount of water absorbed per square foot for comparison.

This experiment was conducted using a randomized complete block design with three replications.

	So-Dri	Brawny	Sparkle	7 <sup>th</sup> Gen	Scott	Bounty	Viva	School
Trial 1	1.071	1.423	0.970	1.505	1.408	2.142	2.392	0.673
Trial 2	1.190	1.328	1.164	0.903	1.267	2.023	2.392	0.337
Trial 3	1.309	1.160	1.26	1.053	1.408	2.023	2.517	0.167

Response: Water

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	9	8.1819234	0.909103	31.3341	<.0001
Brand	7	8.1187296	1.159812	39.9756	<.0001
Trial	2	0.0631938	0.031597	1.0891	0.3634
Error	14	0.4061842	0.029013	Prob>F	
C Total	23	8.5881076			

Brand	Mean	Trial Level	Mean
So-Dri	1.19000	1	1.44800
Brawny	1.30367	2	1.32550
Sparkle	1.13133	3	1.36213
7 <sup>th</sup> Gen	1.15367		
Scott	1.36100		
Bounty	2.06267		
Viva	2.43367		
School	0.39233		

There was a significant difference in brand, but the blocking variable of trial was not significantly effective. The value of Fisher's *LSD* is  $2.145\sqrt{\frac{2(0.029013)}{3}} = 0.2983$ .

Any means that differ by more than 0.3 are considered significantly different. As in the previous experiment, Viva and Bounty have significantly higher absorption/area scores than the other towels. In this experiment, Viva is significantly higher and the school towel has significantly lower mean scores than all other brands.

**Experiment #8a,b**

*Experimental Protocol:* Lay towel flat on a tray. Pour 9 tablespoons of water onto the center of the towel. Hold the towel over the tray for 15 seconds. Measure the amount of the water that has fallen onto the tray to compute the number of tablespoons absorbed by the towel in tablespoons. Also measure the size of the towel and convert these absorption volumes to tablespoons/area.

This experiment was conducted using a randomized complete block design with 4 repetitions. Four brands of towel were used. The data and computer output follow:

	Brawny		Scott		So-Dri		Bounty	
	Vol	Vol/Area	Vol	Vol/Area	Vol	Vol/Area	Vol	Vol/Area
Round 1	7.2	0.047	5.9	0.058	5.25	0.043	6.6	0.056
Round 2	5.25	0.036	3.5	0.038	4.9	0.041	6	0.050
Round 3	6	0.040	3.2	0.031	4.8	0.040	6.1	0.051
Round 4	6	0.040	4.1	0.040	4.6	0.038	6.1	0.051

Response: Volume of Water

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	6	16.021250	2.67021	10.2127	0.0014
Brand	3	11.593125	3.86438	14.7801	0.0008
Round	3	4.428125	1.47604	5.6454	0.0187
Error	9	2.353125	0.26146		
C Total	15	18.374375			

Brand		Round	
Level	Mean	Level	Mean
Brawny	6.11250	1	6.23750
Scott	4.17500	2	4.91250
So-Dri	4.88750	3	5.02500
Bounty	6.20000	4	5.20000

When comparing volume of water (in tablespoons), there is a significant brand effect and a significant round effect. For both brands and rounds, the value of Fisher's

*LSD* is  $2.262\sqrt{\frac{2(0.26146)}{4}} = 0.8179$ . Brawny and Bounty have significantly higher mean

volumes absorbed than do Scott and So-Dri. Brawny and Bounty are not significantly different from each other in regards to average volume of water absorbed. Neither are Scott and So-Dri. For some reason, Round 1 has significantly larger mean volumes absorbed than do Rounds 2, 3, and 4. This may be due to some subtle change in the manner in which the experiment was conducted or the measurements made.

Response: Volume/Area

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	6	0.00063050	0.000105	4.6531	0.0200
Brand	3	0.00033425	0.0001114	4.9336	0.0270
Round	3	0.00029625	0.0000988	4.3727	0.0369
Error	9	0.00020325	0.000023		
C Total	15	0.00083375			

Brand		Round	
Level	Mean	Level	Mean
1	0.040750	1	0.051000
2	0.041750	2	0.041250
3	0.040500	3	0.040500
4	0.052000	4	0.042250

When compared on a volume/area basis, the results are slightly modified. Again there is a significant difference both in brands and rounds. *Fisher's LSD* in this situation

is  $2.262 \sqrt{\frac{2(0.000023)}{4}} = 0.00767$ . As before, Round 1 has a higher mean score than all other rounds. However, on a volume/area basis, Bounty is significantly different from all others, which are not significantly different from each other. The large volume held by Brawny in the previous experiment was possibly due to its large size.

**Experiment #8b**

*Experimental Protocol:* Put one cup of water in a measuring cup. Take a 20 cm by 20 cm section of towel and submerge in the water for 5 seconds. Remove the towel and let it drip for 30 seconds back into the measuring cup. Measure the amount of water removed from the measuring cup to measure the amount of water retained in the towel in fluid ounces. This experiment was set up as a randomized complete block design with 2 replications. The treatment has 7 levels, using 7 kinds of towels.

	Bounty	Scott	Sparkle	So-Dri	Viva	Brawny	7 <sup>th</sup> Gen
Trial 1	1.8	1.5	1.55	1	2	2.18	1
Trial 2	2	1.05	1.73	1.2	2	1.5	1

Response: Water

Analysis of Variance

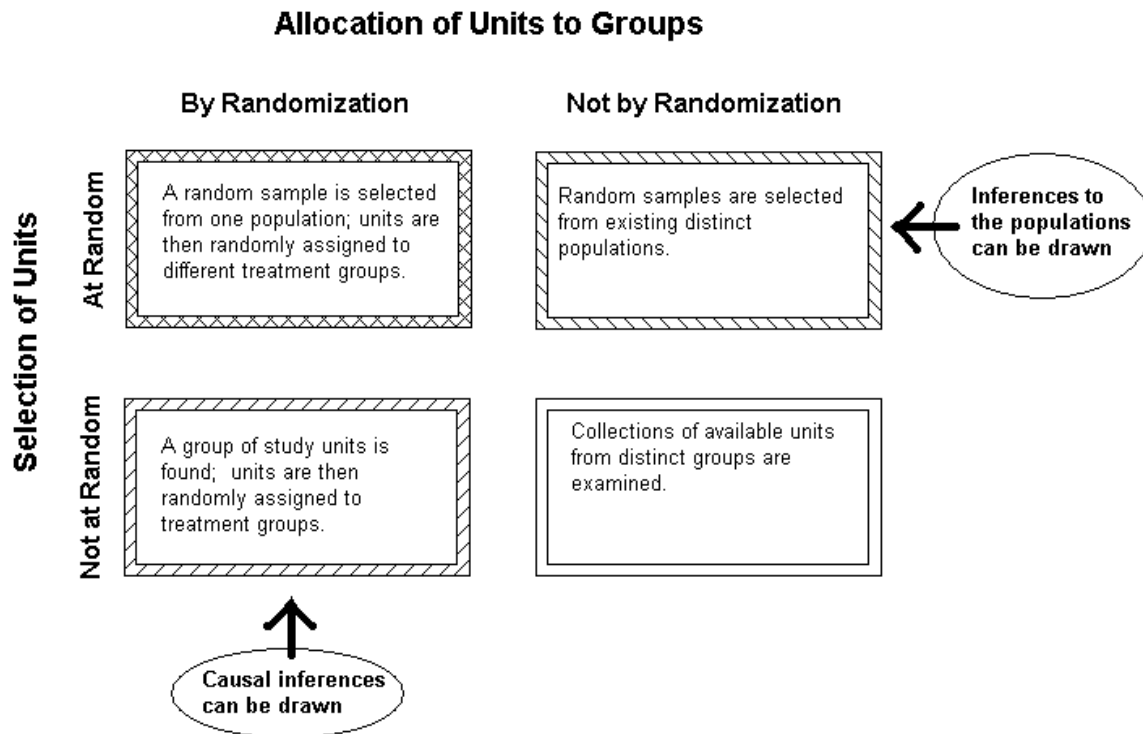
Source	DF	Sum of Squares	Mean Square	F Ratio	Prob>F
Model	7	2.0146786	0.287811	4.7048	0.0390
Brand	6	1.9930714	0.33218	5.4301	0.0294
Trial	1	0.0216071	0.02161	0.3532	0.5740
Error	6	0.3670429	0.061174		
C Total	13	2.3817214			

Towel		Trial	
Level	Mean	Level	Mean
Bounty	1.90000	1	1.57571
Scott	1.27500	2	1.49714
Sparkle	1.64000		
So-Dri	1.10000		
Viva	2.00000		
Brawny	1.84000		
7 <sup>th</sup> Gen	1.00000		

There is a significant effect of brand in mean volume of water retained by the towel. There is no significant effect of the blocking variable Trial. A completely randomized design would be sufficient for this experiment were it to be repeated. For brand, the value of Fisher's *LSD* is  $2.447\sqrt{\frac{2(0.061174)}{2}} = 0.605$ . Differences in mean volume retained larger than 0.61 are considered significant. Bounty and Viva have significantly larger mean volume retained than do Scott, So-Dri, and 7<sup>th</sup> Generation. Bounty, Viva, Brawny, and Sparkle brands are indistinguishable.

**Randomization, Population of Inference, and Causality**

Statistical experiments involve two kinds of randomization, a random assignment of treatment to experimental units and a random selection of experimental units from a population. As shown in the diagram from Ramsey and Schafer's *Statistical Sleuth*, this creates four possible scenarios for an experiment.



Ramsey, Fred L. and Daniel W. Schafer. 1997. *The Statistical Sleuth: A Course in Methods of Data Analysis*. Duxbury Press: Belmont CA.

In these experiments, it is not possible to randomly assign a treatment to an experimental unit or to allocate a unit to a group. A Viva towel is a Viva towel, and you cannot make it a Bounty towel. The groups are already established. In terms of Ramsey and Schafer's diagram, the experimenter is in the second column. We can make no statements of causality based on our experimental results. We can only describe the differences in strength and absorption that we see.

The population of inference to which our observed results apply depends on the method of sampling from the population. The population of inference is always the population from which the experimental units were selected at random. Four possible scenarios are considered below:

- (1) The first four towels from a roll are used in the experiment.  
Here there is no random selection of experimental units. Thus, there is no population of inference and no inference is possible.
- (2) Towels are randomly selected from a roll.  
Here the population of inference is the roll from which the towels were selected. Inferences can be made about the response characteristics of other towels on that roll, but not other towels on other rolls.
- (3) Towels were selected at random from different rolls which were purchased at the same store.  
We assume that the rolls were selected at random from those available in the store. The population of inference is the lot of towels from which the towels selected come. There is always variability in the production process, and towels from the same lot will be less variable than towels from different lots. Inferences can be made concerning the mean response of other towels from these lots, but not to towels from other lots or to the population of towels of that brand.
- (4) Towels were selected at random from different rolls purchased at different stores.  
Because the towels were presumably taken from different lots, we can believe that the population of inference is the towels of that brand. Of course, having more towels from different locations would increase our confidence in getting a sample of towels from each manufacturer. If at all possible, towels should be purchased at different stores in different locations.