Appendix B: Partitioning the Sums of Squares

The deviation of any single observation, y_{ti}, from the mean of all observations, $\bar{y}_{\cdot\cdot}$, can always be partitioned into two components. In the terminology of Wannacott and Wannacott, the

total deviation = explained deviation + unexplained deviation.

$$y_{ti} - \bar{y}_{\cdot\cdot} = (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot}) + (y_{ti} - \bar{y}_{t\cdot}).$$

The equation is obviously true, since we have added then subtracted $\bar{y}_{t\cdot}$ to the left side of the equation.

Now, square both sides of the equation and sum over *t* and *i*.

$$\sum_t \sum_i (y_{ti} - \bar{y}_{\cdot\cdot})^2 = \sum_t \sum_i \left[(\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot}) + (y_{ti} - \bar{y}_{t\cdot}) \right]^2$$
$$= \sum_t \sum_i \left[(\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})^2 + 2(\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})(y_{ti} - \bar{y}_{t\cdot}) + (y_{ti} - \bar{y}_{t\cdot})^2 \right]$$
$$= \sum_t \sum_i (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})^2 + 2\sum_t \sum_i (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})(y_{ti} - \bar{y}_{t\cdot}) + \sum_t \sum_i (y_{ti} - \bar{y}_{t\cdot})^2 .$$

Simplify the middle term in this expansion to show that it is always zero.

$$2\sum_t \sum_i (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})(y_{ti} - \bar{y}_{t\cdot}) = 2\sum_t \left[(\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot}) \sum_i (y_{ti} - \bar{y}_{t\cdot}) \right]$$

We know that the sum $\sum_i (y_{ti} - \bar{y}_{t\cdot}) = 0$ since this is just the total variation from the mean, which is always zero.

Also note that $\sum_t \sum_i (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})^2 = n_t \sum_t (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})^2$, since the summand is independent of the subscript *i*.

So, we can now rewrite $\sum_t \sum_i (y_{ti} - \bar{y}_{\cdot\cdot})^2 = n_t \sum_t (\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot})^2 + \sum_t \sum_i (y_{ti} - \bar{y}_{t\cdot})^2$. This equation states that the

Total Sums of Squares = Treatment Sums of Squares + Residual Sums of Squares

Reference:

Wannacott, Thomas H. and Ronald J. Wannacott, *Introductory Statistics*, John Wiley and Sons, Inc. New York, New York, 1969.