

*Investigations of sample statistic distributions using an on-line simulator*

0. Go to the following web site: <http://onlinestatbook.com/rvls.html>  
 Go to **Simulations/Demonstrations**, then select **Sampling Distribution Simulation**, then click on the button at the top of the left column labeled **Begin**.  
 You will see 4 sets of axes. For the instructions below, consider the uppermost one Graph 1 and number down so that Graph 4 is the lowest.
- 1a. Use a normal distribution for graph 1 and make graph 3 show a distribution of sample means when sample size is 2. Click on Animated next to graph 2 and watch to see what the simulation is doing. Then make a graph of the more complete distribution by doing several thousand simulations.
- 1b. What is the mean of the population? What is the standard deviation of the population? (These numbers are shown to the left of the population graph.) What is the mean of the sample means? What is the standard deviation of the sample means? (These numbers are shown to the left of the graph of the distribution of the sample means.)
- 1c. Continue using the normal population distribution, but change your sample size from 2 to 5. What happens to the standard deviation of the sample means now? Complete the following table:  
 Note the population standard deviation: \_\_\_\_\_

Sample size $n$	2	5	10	16	25
s.d. of sample mean					

Check that your simulations support the relationship you learned between the standard deviation of the sample mean and the standard deviation of the population and sample size.

2. Repeat the above steps for the uniform distribution, the skewed distribution, and a bimodal custom distribution. Verify that the standard deviation of the means relationship holds true for these other distributions.

Uniform distribution: Note the population standard deviation: \_\_\_\_\_

Sample size $n$	2	5	10	16	25
s.d. of sample mean					

The given skewed distribution: Note the population standard deviation: \_\_\_\_\_

Sample size $n$	2	5	10	16	25
s.d. of sample mean					

Your bimodal distribution: : Note the population standard deviation: \_\_\_\_\_

Sample size $n$	2	5	10	16	25
s.d. of sample mean					

3a. In this question, you are to pay attention to the *shape* of the distribution of the sample mean. Use the skewed distribution for your population. Look at the distribution of the sample means when  $n = 2$ ,  $n = 5$ , and so on. What happens to the shape of the distribution of the sample means as  $n$  increases?

3b. Repeat this for the uniform distribution, and for a bimodal distribution. Does the observation you made in part 3a still hold true?

4a. In this question, you will be comparing the distribution of a sample *mean* to the distribution of a sample *median*. Begin with a normal distribution for graph 1. Make graph 3 show the distribution of the sample *mean* with sample size  $n = 5$ , and make graph 4 show the distribution of the sample *median* with sample size  $n = 5$ . Animate slowly at first, then complete the distribution. What do you observe about the center of the distribution of

the sample means?

the sample medians?

4b. Which has more spread: the distribution of the sample means or the distribution of the sample medians? Which would be a more *precise* predictor of the center of this population?

4c. Continuing with the normally distributed population in graph 1, increase sample sizes for means and medians to  $n = 25$ . Do the observations you made in parts 4a and 4b still hold true?

5a. In this question you will be comparing the distribution of a sample variance calculated with division by  $n$  to the distribution of a sample variance calculated with division by  $n-1$ . Begin with the normal population in graph 1. Note that the population variance equals  $5^2 = 25$ . Make graph 3 show the distribution of the variance with sample size  $n = 5$ . (This is the value calculated with division by  $n$ .) Make graph 4 show the distribution of the  $\text{Var}(U)$  with sample size  $n = 5$ . (This is the value calculated with division by  $n-1$ .) Animate one simulation at a time and be sure you understand what you are seeing. Which sample variance is larger? Why?

5b. Once you are comfortable with what you are seeing, show the distribution of each variance using several thousand simulations. What is the mean of each distribution of sample variances?

Mean when divide by  $n =$

Mean when divide by  $n-1 =$

5c. Compare the mean of each sampling distribution to the population variance. Which sample statistic appears to be an *unbiased estimator* of the population variance?