Scope of Inference

Suppose we want to determine if, when we spin a penny, the probability of obtaining a head from a spin of a coin differed from 0.5. Every action one takes in conducting an experiment affects the analysis and/or conclusions from the study. In this case, I always use the same 1999 penny. What impact does that have on my conclusions? Can I draw inference about any other penny?

The scope of inference refers to the population to which inference can reasonably be drawn based on the study. This population is the population from which the random sample used in the study was drawn. If only one penny was used, we consider the results a random sample of results possible with this penny. We can comment only on this penny. If someone suggests that our penny is special and other pennies wouldn't spin similarly, we have no counter-argument. We only have information about that one penny.

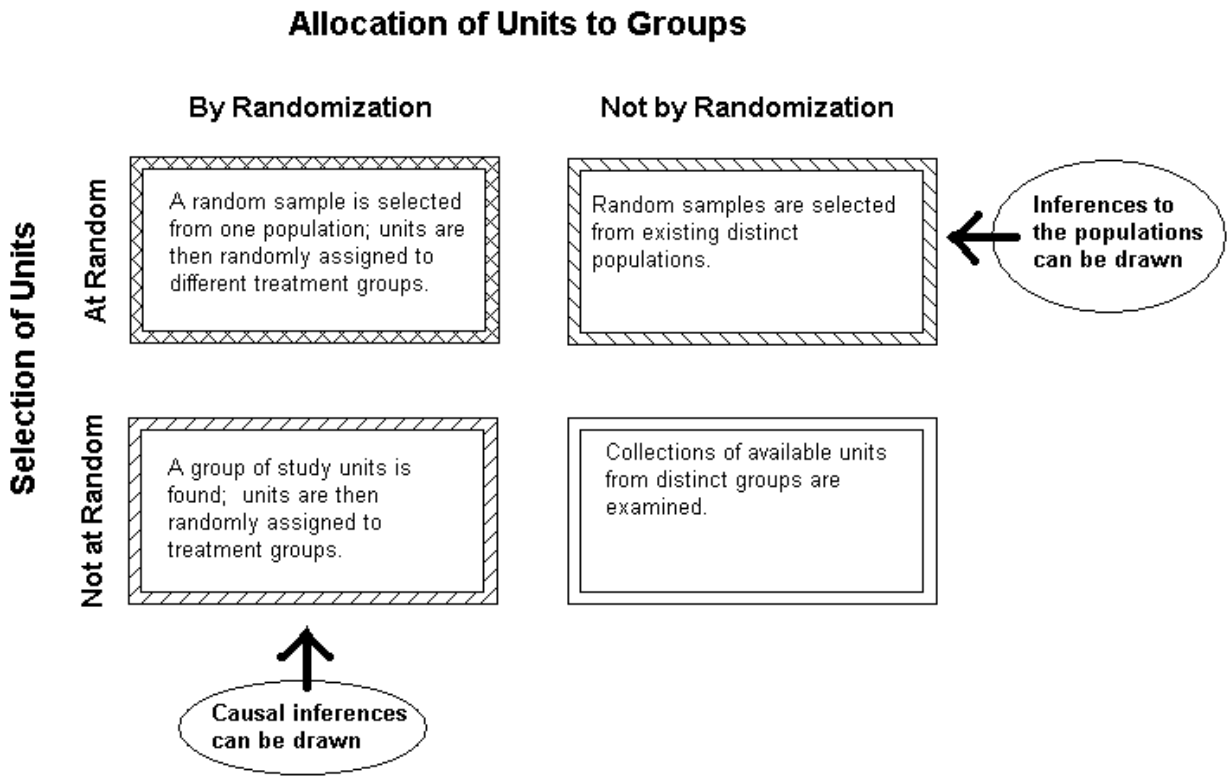
What are the advantages to increasing the scope of inference? What are the advantages to decreasing the scope of inference?

If we had taken a random sample of pennies from a collection of 1999 pennies, we introduce the variation inherent in those pennies. Some are worn more than others, may be nicked or otherwise altered. This variation makes it more difficult for us to find a difference if one exists. However, if we do find a significant difference from $p = 0.5$, we can say something about 1999 pennies, not just one special penny.

If we had taken a random sample from pennies of all ages, even more variation is put into the system. The more variation, the more difficult to achieve a significant result, but if we do, we can make statements about the spinning probabilities of pennies of all years, not just 1999 pennies or a single penny.

Randomization and Inference

The diagram below from Ramsey and Schafer (1997) illustrates the essential role randomization plays in the scope of statistical inference. There are four fundamental compartments in the diagram below formed by partitioning the Selection of Units and the Allocation of Units to Group into two distinct groups, namely, *By Randomization* and *Not by Randomization*.



Ramsey, Fred L. and Daniel W. Schafer. 1997. *The Statistical Sleuth: A Course in Methods of Data Analysis*. Duxbury Press: Belmont CA.

Suppose a dentist wants to know if a daily dose of 500 mg of vitamin C will result in fewer canker sores in the mouth than taking no vitamin C.

We will consider 4 scenarios that correspond to the 4 divisions shown above.

Case 1) No Randomization in Selection of Units and No Randomization in Allocation of Units to Treatments

The dentist, working through the local dental society, convinces all of the dental patients in town with appointments the first two weeks in December to be subjects in an experiment. He divides them into two groups, those who take at least 500 mg of vitamin C each day and those who don't. He then asks them how often they have canker sores in

their mouth and checks their patients records to see who has complained about canker sores. He compares the proportion of those who take vitamin C daily and complain of canker sores with the proportion of those who don't take vitamin C and complain of canker sores. There is a significant difference in the two proportions, with a significantly smaller proportion of those taking vitamin C having canker sores. What can we conclude?

Case 2) No Randomization in Selection of Units but Randomization in Allocation of Units to Treatments

A dentist, working through the local dental society, convinces all of the dental patients in town with appointments the first two weeks in December to be subjects in an experiment. He randomly assigns half of them to take 500 mg of vitamin C each day and the other half to abstain from taking vitamin C for three months. At the end of this time he determines the proportion of each group that has suffered from canker sores during those three months. There is a significant difference in the two proportions, with a significantly smaller proportion of those taking vitamin C having canker sores. What can we conclude?

Case 3) Randomization in Selection of Units but No Randomization in Allocation of Units to Treatments

The dentist, working through the local dental society, selects a random sample of dental patients in town and convinces them to be subjects in an experiment. He divides them into two groups, those who take at least 500 mg of vitamin C each day and those who don't. He then asks them how often they have canker sores in their mouth and checks their patients records to see who has complained about canker sores. He compares the proportion of those who take vitamin C daily and complain of canker sores with the proportion of those who don't take vitamin C and complain of canker sores. There is a significant difference in the two proportions, with a significantly smaller proportion of those taking vitamin C having canker sores. What can we conclude?

Case 4) Randomization in Selection of Units and Randomization in Allocation of Units to Treatments

The dentist, working through the local dental society, selects a random sample of dental patients in town and convinces them to be subjects in an experiment. He randomly assigns half of them to take 500 mg of vitamin C each day and the other half to abstain from taking vitamin C for three months. At the end of this time he determines the proportion of each group that has suffered from canker sores during those three months. There is a significant difference in the two proportions, with a significantly smaller proportion of those taking vitamin C having canker sores. What can we conclude?

Conclusions

Case 1) Since the patients do not represent a random sample from any population, it is not possible to make any inference about this result holding for a larger population. Since the study was observational, with subjects not randomly assigned to treatments, no causal inference can be made. We just know that for these patients, those who take vitamin C have fewer canker sores than those who don't. We don't know why, and we don't know if this result would be consistent with another group.

Case 2) Since the patients do not represent a random sample from any population, it is not possible to make any inference that this result would hold for a larger population. However, the treatments were randomly assigned to the subjects, so (assuming other factors were controlled or randomized) the difference in proportions having canker sores can be attributed to the vitamin C. We don't know if this result would be consistent with another group, but we believe we know why, for this group, the proportions differ.

Case 3) Since the patients selected were a random sample of dental patients in town, we can infer that the results observed in this experiment would be consistent with results from the whole population of dental patients in this town. However, since the study was observational, with subjects not being randomly assigned to treatments, no causal inference can be made. We believe that for the population of dental patients in this town, that those taking vitamin C have fewer canker sores than those who didn't. We don't know if it is the vitamin C that causes this reduction or some other confounding variable. We cannot conclude that for the general population, those taking vitamin C have fewer canker sores, since the sample was only of dental patients. To the extent that the dental patients in this town are representative of dental patients in general, we can infer that dental patients who take vitamin C tend to have fewer canker sores than those who don't.

Case 4) Since the patients selected were a random sample of dental patients in town, we can infer than the results observed in this experiment would be consistent with results from the whole population of dental patients in this town. Moreover, the treatments were randomly assigned to the subjects, so (assuming other factors were controlled or randomized) the difference in proportions having canker sores can be attributed to the vitamin C. We believe that for the population of dental patients in this town, that those taking vitamin C have fewer canker sores than those who don't. Also, we believe that the reduction in canker sores is a consequence of taking the vitamin C. We cannot conclude that for the general population, those taking vitamin C have fewer canker sores, since the sample was only of dental patients. To the extent that the dental patients in this town are representative of dental patients in general, we can infer that dental patients who take vitamin C tend to have fewer canker sores than those who don't, as a result of taking the vitamin C.