Wind Chill Table Investigation

Dan Teague, NC School of Science and Mathematics

The standard wind-chill table from an almanac provides a challenging investigation in re-expression and curve fitting. The table is presented below:

Wind (mph)	**Temperature in degrees Farhenheit** (Winds geater than 45 mph have little additonal cooling effect)														
	35	**30**	**25**	**20**	**15**	**10**	**5**	**0**	**-5**	**-10**	**-15**	**-20**	**-25**	**-30**	**-35**
5	33	27	21	16	12	7	0	-5	-10	-15	-21	-26	-31	-36	-42
10	22	16	10	3	-3	-9	-15	-22	-27	-34	-40	-46	-52	-58	-64
15	16	9	2	-5	-11	-18	-25	-31	-38	-45	-51	-58	-65	-72	-78
20	12	4	-3	-10	-17	-24	-31	-39	-46	-53	-60	-67	-74	-81	-88
25	8	1	-7	-15	-22	-29	-36	-44	-51	-59	-66	-74	-81	-88	-96
30	6	-2	-10	-18	-25	-33	-41	-49	-56	-64	-71	-79	-86	-93	-101
35	4	-4	-12	-20	-27	-35	-43	-52	-58	-67	-74	-82	-89	-97	-105
40	3	-5	-13	-21	-29	-37	-45	-53	-60	-69	-76	-84	-92	-100	-107
45	2	-6	-14	-22	-30	-38	-46	-54	-62	-70	-78	-85	-93	-102	-109

Consider each row in the table. Obviously, the values are rounded to the nearest degree. The wind-chill temperatures for a wind of 5 mph gives the following set of data.

Actual Temp	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35
Wind-Chill Temp	33	27	21	16	12	7	0	-5	-10	-15	-21	-26	-31	-36	-42

Students should recognize that as the Actual Temperature is decreased by 5 degrees, the Wind-Chill Temperature decreases by either 5 or 6 degrees. Thus, a linear model will represent this data well.

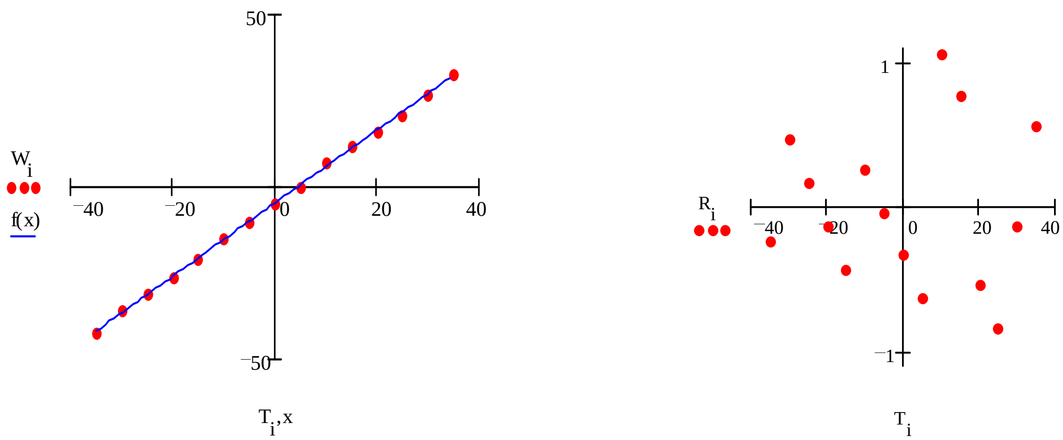

Figure 22: Linear Model for 5 mph Wind

Students may note a sawtooth aspect to the residual plot. This is characteristic of data that has been rounded. Not only is this set of data linear, but the corresponding data for each wind velocity is linear. This is a good practice set for entering data and fitting linear models.

For each wind velocity, we can find the linear model. Here is the list:

Wind Velocity	Linear Least-Squares Model
5 mph	$C_5 = 1.058T - 4.650$
10 mph	$C_{10} = 1.24T - 21.30$
15 mph	$C_{15} = 1.343T - 31.34$
20 mph	$C_{20} = 1.425T - 38.46$
25 mph	$C_{25} = 1.479T - 43.90$
30 mph	$C_{30} = 1.521T - 48.10$
35 mph	$C_{35} = 1.551T - 50.71$
40 mph	$C_{40} = 1.573T - 52.49$
45 mph	$C_{45} = 1.586T - 53.76$

As the wind velocity changes, the slopes and intercepts of the regression lines change. This means the slopes and intercepts are a function of the wind velocity. It appears that the wind-chill temperature can be written as $C = f(w)T + g(w)$ for some functions *f* and *g*. The data defining functions f and g are given below:

Wind Velocity	5	10	15	20	25	30	35	40	45
Slope	1.058	1.235	1.43	1.25	1.479	1.521	1.551	1.573	1.586
Intercept	-4.650	-21.30	-31.34	-38.46	-43.90	-48.10	-50.71	-52.49	-53.76

The scatterplots for the slopes and intercepts are given below:

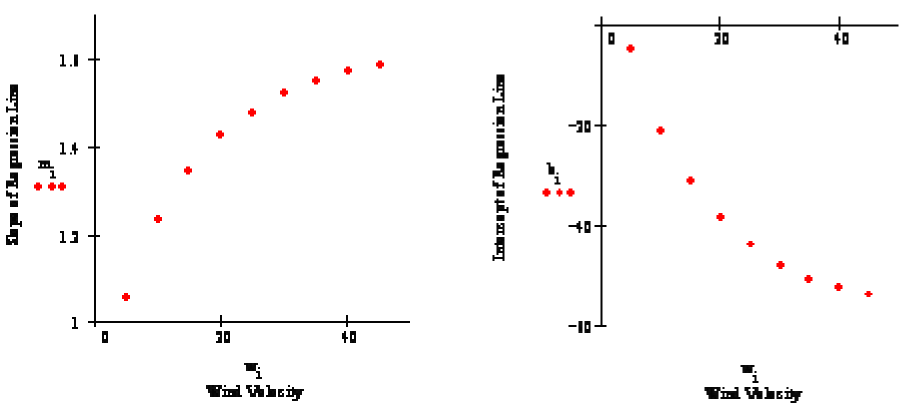

Figure 23: Slope and Intercept Functions

What functions model this behavior?

A statement on the table that wind velocities greater than 45 mph have little additional cooling effect suggests asyptotic behavior. We can try exponential functions since they also illustrate this asymptotic behavior. If the relationship between Wind Velocity and Slope is exponential, what kind of exponential function do we have? We can argue that an exponential function of the form $m = -Ae^{-kw} + B$ might fit. From our graph, we can consider an initial guess of $B = 1.7$. If this is correct, then the re-expression $(w, \ln(1.7 - m))$ should be linear. The residual plot indicates this is not linear. The re-expression $(w, \ln(1.6 - m))$ is then tried and we see that this residual plot has the opposite curvature. There is a value between 1.6 and 1.7 that shoud work.

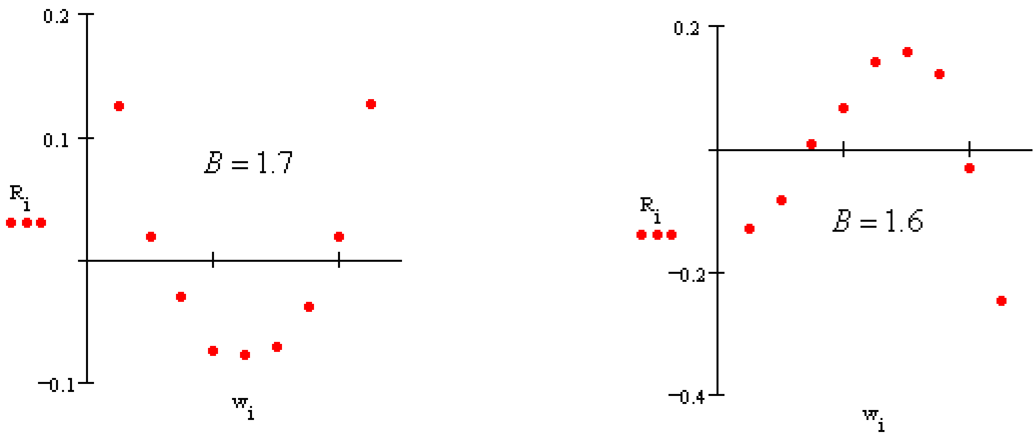

Figure 24: Residual Plots Indicating Change in Concavity

By trial and error, we find 1.623 is a good value.

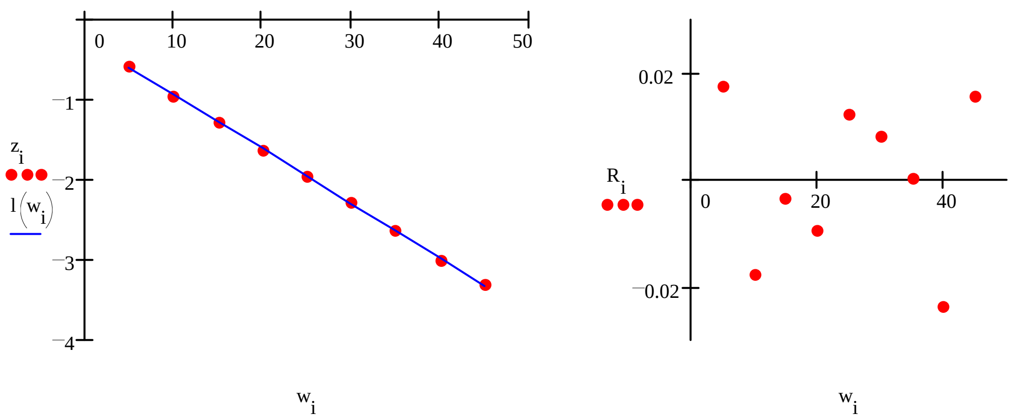

Figure 25: Linearized Data

The model, then is, $f(w) = 1.623 - 0.78e^{-0.068w}$.

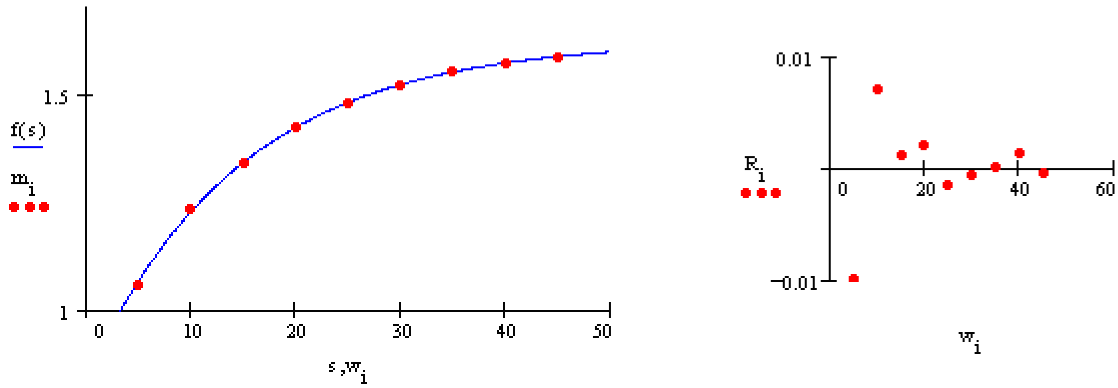

Figure 26: Exponential Model with Residuals

With similar techniques, we can show that the relationship between the wind velocity and the intercept of the regression line is $g(w) = 70.316e^{-0.66w} - 57.5$. So the wind-chill function can be defined as

$$C(w,T) = \left(1.623 - 0.78e^{-0.068w}\right) \cdot T + \left(70.316e^{-0.66w} - 57.5\right).$$

The numbers in the wind-chill table are the values of this function rounded to the nearest integer.