

Violating Conditions in Significance Testing (Student Materials)

Julie Hicks, Woodbridge High School, Woodbridge, VA
jjhicks@home.com

John Lieb, The Roxbury Latin School, West Roxbury, MA
jlieb19@yahoo.com

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The goal of the following activity is to provide students with a more concrete idea of what it means for a test of significance to be *robust*. Students will need the TI-83 program **ROBUSTT** when completing this activity.

See **Teacher Notes** for a more detailed explanation of this activity and the ideas behind them.

Contents

The worksheets in this document should be completed before doing simulations with the calculator program **ROBUSTT**.

- **Student worksheet – “Violating a condition of the t -test for one mean”**
 - **8 pages**

- **Data recording worksheet – t -test for the population mean**
 - **1 page**

- **Student worksheet – “Violating conditions of the t -test for the population slope”**
 - **10 pages**

- **Data recording worksheet – t -test for the population slope**
 - **1 page**

Violating a condition of the t -test for one mean

Suppose you want to learn about a numeric characteristic (*e.g.* age, cholesterol level, income) of a population. Would you perform a t -test on sample data **if** you *knew* the value of the population mean? The answer, of course, is **NO**. The whole point of inference is to learn about a population parameter – if we already know the population mean, then there is no reason to do a t -test.

But, in the following activities you will perform t -tests when you *know* the value of the population mean. Why would you do this? The answer will become clear after doing the following activity, but let us offer a legitimate reason for doing this: It will help you understand what it means for the t -test for one mean to be called **robust**.

One of the necessary conditions for the t -test for the population mean is

- The population from which the sample is taken must be normally distributed.

Let us refer to the above condition briefly as the “**normal population**” condition of the t -test for the population mean. What happens when this condition is not true? Textbooks say that the t -test is **robust** to violations of this condition. But what does **robust** mean? To what does it refer? Understanding robustness is the goal of this activity.

The following activity simulates data that violate the **normal population** condition of the t -test for the population mean and you will examine the consequences. This activity is divided into three parts. At the end of each part you will use the calculator program **ROBUSTT** to simulate the process you worked through by hand.

Here are four questions you need to answer before proceeding.

- Suppose a t -test has a significance level of α . How does the value of α affect the decision to reject or fail to reject the null hypothesis?
- What is a “Type I error” in the context of tests of significance?
- How does the significance level α relate to the probability of making a Type I error?
- When doing a t -test, what is the potential drawback to making α very small ($\alpha = 0.01$ or $\alpha = 0.005$)?

PART I
Condition satisfied – *The population is normal!*

Before we begin violating the **normal population** condition, we will examine a situation when the conditions for the t -test are satisfied *exactly*.

Consider a population that is **normally distributed** with $m = 50$ and $s = 10$. Knowing these facts about our population, let us consider what would happen if we performed a test of significance.

If we actually know the population mean is 50 and perform a t -test for the population mean, how often would our decision be wrong? Specifically, how often will we reject the null hypothesis when it is, in fact, true? Let us test the null hypothesis

$$H_o : \mu = 50 \quad \text{vs.} \quad H_A : \mu \neq 50$$

1. Use your calculator to generate 5 observations from a population that is **normally distributed** with $m = 50$ and $s = 10$ using the command below. The 5 observations will be in list L1.

`randNorm(50, 10, 5) @ L1`

2. Fill in the table below with your observations. Round to three decimal places.

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3. Use your calculator to calculate the sample mean and sample standard deviation of your 5 observations and fill them in below to two decimal places.

$$\bar{x} = \qquad s =$$

4. Calculate the t statistic for your sample mean by hand using the formula below.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

5. Use your calculator or the t distribution table to calculate the p -value for your sample. Record the p -value below.

6. Suppose we used a significance level of $\alpha = 0.10$. (Note that this is a higher significance level than would normally be used in practice. We are using an α this large so that your class will see some rejections of the null hypothesis. An $\alpha = 0.01$ would not generate many rejections. Why?) Compare your p -value to the significance level and make a decision about the null hypothesis.

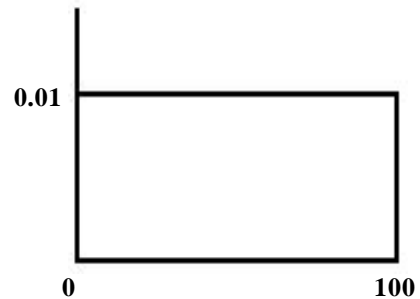
Now, note that the sample data were generated by assuming the value of the null hypothesis mean was our population mean. Thus, in this case we *know* the population mean equals the value of the null hypothesis mean, $m = 50$. If you rejected the null hypothesis, then you *know* that you have made a Type I error! If you failed to reject the null hypothesis, then you made a correct decision. (Recall that *in practice*, since the population mean is *unknown*, you cannot be certain whether you are making a Type I error or not – only that a Type I error occurs 10% of the time when the null hypothesis is true.)

7. The calculator program **ROBUSTT** automates the process you just went through. In the program you set the significance level α and the sample size n . Right now let $\alpha = 0.10$ and $n = 5$. Use the data recording worksheet to record the results of running the program 25 times. Be sure to record your results in the appropriate column.
8. After simulating 25 t -tests using the calculator program, how many times did you reject the null hypothesis? Is this more or less than the number you would expect?
9. Report your results to your teacher so they can be combined with other student results. Ideally, the rate at which your class rejects the null hypothesis should equal $\alpha = 0.10$. Why?

Next, you will examine a case when the **normal population** condition is *not* satisfied.

PART II
Condition violated – Uh-oh, the population is non-normal!

Now, data will be selected from the population of all real numbers between 0 and 100 inclusive. Data will be rounded to two decimal places and all possible values are equally likely. Thus, 56.78 has the same chance of being picked as 4.31. Such a population has a *uniform* distribution with a lower bound of 0 and an upper bound of 100. Not surprisingly, the mean of this population is 50. The density curve for this distribution is shown at right.



If we actually know the population mean is 50 and perform a *t*-test for the population mean, how often would our decision be wrong? Specifically, how often will we reject the null hypothesis when it is, in fact, true? We will test the null hypothesis

$$H_0 : \mu = 50 \quad \text{vs.} \quad H_A : \mu \neq 50$$

➤ **Note that we are violating one of the conditions of the *t*-test. The population is *not* normally distributed!**

10. Use your calculator to generate 5 observations from a population that is uniformly distributed between 0 and 100 by using the command below. `rand` generates a random number between 0 and 1 – it is found on the calculator by going to MATH ® PRB. The 5 numbers will be stored in list L1.

`100rand®L1`

11. Fill in the table below with your observations. Round to three decimal places.

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12. Use your calculator to calculate the sample mean and sample standard deviation of your 5 observations and fill them in below to two decimal places.

$$\bar{x} = \qquad s =$$

13. Calculate the *t* statistic for your sample mean by hand using the formula:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

14. Use your calculator or the t distribution table to calculate the p -value for your sample. Record the p -value below.

15. Suppose we used a significance level of $\alpha = 0.10$. (Again, note that this is a higher significance level than would normally be used in practice.) Compare your p -value to the significance level and make a decision about the null hypothesis.

Once again note that the sample data were generated by assuming the value of the null hypothesis mean was our population mean. Thus, in this case we **know** the population mean equals the value of the null hypothesis mean, $m = 50$. If you rejected the null hypothesis, then you **know** that you have made a Type I error! If you failed to reject the null hypothesis, then you made a correct decision.

16. The calculator program **ROBUSTT** automates the process you just went through. In the program you set the significance level α and the sample size n . As in Part I, let $\alpha = 0.10$ and $n = 5$. Use the data recording worksheet to record the results of running the program 25 times. Be sure to record your results in the appropriate column.

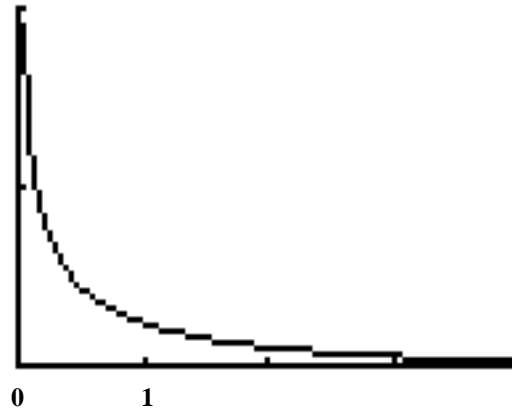
17. After simulating 25 t -tests using the calculator program, how many times did you reject the null hypothesis? Is this more or less than the number you would expect?

18. Report your results to your teacher so they can be combined with other student results. **If** the t -test is **robust** when the population is uniform and $n = 5$, the probability of incorrectly rejecting the null hypothesis – making a Type I error – should be close to the stated significance level α . What do the results of your class show?

Finally, you will examine a case where the **normal population** condition is violated more severely.

PART III
Condition violated – Oh no! The population is really non-normal!

You will go through the process from Parts I and II with one more population. This population is strongly skewed to the right and possesses a density distribution that looks like the picture at right. It is called a *chi-square distribution with one degree of freedom*. This distribution takes values greater than 0 and has a mean of 1. (The name of this distribution is not terribly important -- all you really need to know is we have a strongly skewed right population with a mean of 1.)



If we actually know the population mean is 1 and perform a *t*-test for the population mean, how often would our decision be wrong? Specifically, how often will we reject the null hypothesis when, in fact, it is true? Let us test the null hypothesis

$$H_o : \mu = 1 \quad \text{vs.} \quad H_A : \mu \neq 1$$

- **Note that we are violating one of the conditions of the *t*-test. The population is *not* normally distributed – it is strongly skewed to the right!!**

We will use the calculator to generate 5 observations from a chi-square population with one degree of freedom.

19. Unfortunately, the calculator does not have a randChi -Sq button, but we can still use the random number features of the calculator to randomly sample from a chi-square distribution. (How? See below if you are interested in the details.)¹ The following command picks 5 random numbers from the chi-square distribution pictured above and stores them in list L1.

$$(\text{randNorm}(0, 1, 5))^2 \text{ @ L1}$$

20. Fill in the table below with your observations. Round to three decimal places.

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¹ If we square the normal distribution with a mean of 0 and a standard deviation of 1 (abbreviated $N(0, 1)$), the resulting distribution has a *chi-square distribution with one degree of freedom* (which has a mean of 1). Thus, generating 5 numbers from $N(0, 1)$ and squaring each of them is equivalent to generating 5 numbers from a *chi-square distribution with $df = 1$* .

21. Use your calculator to calculate the sample mean and sample standard deviation of your 5 observations and fill them in below to two decimal places.

$$\bar{x} = \qquad s =$$

22. Calculate the t statistic for your sample mean by hand using the formula below.

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

23. Use your calculator or the t distribution table to calculate the p -value for your sample. Record the p -value below.
24. Suppose we used a significance level of $\alpha = 0.10$. (Again, note that this is a higher significance level than would normally be used in practice.) Compare your p -value to the significance level and make a decision about the null hypothesis.

One more time, note that the sample data were generated by assuming the value of the null hypothesis mean was our population mean. Thus, in this case we *know* the population mean equals the value of the null hypothesis mean, $\mu = 1$. If you rejected the null hypothesis, then you *know* that you have made a Type I error! If you failed to reject the null hypothesis, then you made a correct decision.

25. The calculator program **ROBUSTT** automates the process you just went through. In the program you set the significance level α and the sample size n . As in Parts I and II, let $\alpha = 0.10$ and $n = 5$. Use the data recording worksheet to record the results of running the program 25 times. Be sure to record your results in the appropriate column.
26. After simulating 25 t -tests using the calculator program, how many times did you reject the null hypothesis? Is this more or less than the number you would expect?
27. Report your results to your teacher so they can be combined with other student results. Then read the comments on the next page.

OK, so what was the point of doing all that work?

If a test is **robust**, then violating conditions of the test should not cause the probability of a Type I error to differ from the stated significance level α . The more robust a test is, the less effect violations of the **normal population** condition will have on the probability of a Type I error. Your class has simulated several hundred t -tests when you **knew** that the null hypothesis was true. Thus, you can compare the proportion of times your class rejected the null hypothesis for each of the three population distributions with the stated significance level α .

28. Fill in the table below with the **combined results of your class**. (You will not yet be able to fill in the third column.) What do they indicate about the robustness of the t -test to the non-normal population distributions $\alpha = 0.10$ and $n = 5$?

POPULATION	NORMAL		UNIFORM		R-SKEW TRIANGULAR		CHI-SQ (DF=1)	
	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho
DECISION								
TOTALS								
% REJECTED Ho								

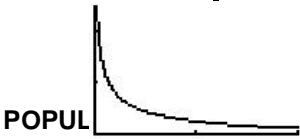
29. Note that you worked through t -tests with **three** known population distributions and that the data recording worksheet and the table above have a fourth one listed. We will call this distribution a **right triangular** distribution. Its density curve is pictured at right. In terms of skewness, consider it a compromise between the **uniform** distribution and the **chi-square** distribution.



The program **ROBUSTT** will simulate t -tests with data simulated from this right-triangular distribution as it does with the other three population distributions. If you have time, use the calculator to generate some results for this population distribution. Record your results on the data recording worksheet.

***t*-test for one population mean**

Alpha =	
Sample Size =	

	NORMAL		UNIFORM		RIGHT TRIANGULAR		CHI-SQUARE (DF=1)	
TRIAL	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
TOTALS								
% REJECTED Ho								

Violating conditions of the t -test for the population slope

Suppose you want to learn about the relationship between two quantitative variables for a population (*e.g.* height and weight for a population of college students). The population slope b of the true regression line would be a parameter that would interest you. Would you perform a t -test on sample data from this population **if you *knew*** the value of the population slope? The answer, of course, is **NO**. The whole point of inference is to learn about a population parameter – if we already knew the population slope, then there is no reason to do a t -test.

But, in the following activities you will perform t -tests for the population slope when we **know** the value of the population slope – in all cases the value of the population slope b will equal 0. Why would we do this? The answer will become clear after doing the following activity, but let us offer a legitimate reason for doing this: It will help you understand what it means for the t -test for the population slope to be called **robust**.

Two of the necessary conditions for the t -test for the population slope are as follows:

- The standard deviation of the errors is constant for all values of x .
- For a fixed value of x , the distribution of the errors around the mean value of y is normally distributed with a mean of 0 and a standard deviation of σ .

Let us refer to the first of the above conditions as the “**constant standard deviation of errors**” condition of the t -test for the population slope. Let us refer to the second of the above conditions as the “**normality of errors**” condition. What happens when these conditions are violated? Textbooks say that the t -test is a robust test. But what does **robust** mean? To what does it refer? Understanding robustness is the goal of this activity.

The following activity will violate the above two conditions and examine the consequences. This activity is divided into three parts. At the end of each part you will use the calculator program **ROBUSTT** to simulate the process you worked through by hand.

Here are four review questions you need to answer before proceeding.

- Suppose a t -test has a significance level of α . How does the value of α affect the decision to reject or fail to reject the null hypothesis?
- What is a “Type I error” in the context of tests of significance?
- How does the significance level α relate to the probability of making a Type I error?
- When doing a t -test, what is the potential drawback to making α very small ($\alpha = 0.01$ or $\alpha = 0.005$)?

PART I

Condition satisfied – *The errors are normally distributed with constant variance*

Before we begin violating conditions, we will examine a situation when the conditions for the t -test for the population slope are satisfied *exactly*. Consider two variables that have no relationship, and therefore a regression line with a population slope \mathbf{b} of zero. Knowing this, let us consider what would happen if we performed a test of significance on the slope \mathbf{b} . How often would we incorrectly reject the null hypothesis when, in fact, it is true? Let us test the null hypothesis

$$H_0 : \mathbf{b} = 0 \quad \text{vs.} \quad H_A : \mathbf{b} \neq 0$$

Now you will generate sample data under the assumption that $\mathbf{b} = 0$. You will generate 8 observations from a population that has a regression line $y = 6$ and an error structure that is normally distributed with $\mu = 0$ and $\sigma = 1$ (hereafter abbreviated as $N(0, 1)$). Thus, our conditions for doing a t -test for the slope are satisfied. Why?

1. The values of the explanatory variable will be integers from 1 to 8. Use the command below on your calculator to create values of an explanatory variable in list L1. The command `seq` can be found under 2^{nd} $\text{\textcircled{R}}$ `STAT` $\text{\textcircled{R}}$ `OPS`. (Or, of course, you can type these numbers into list L1.)

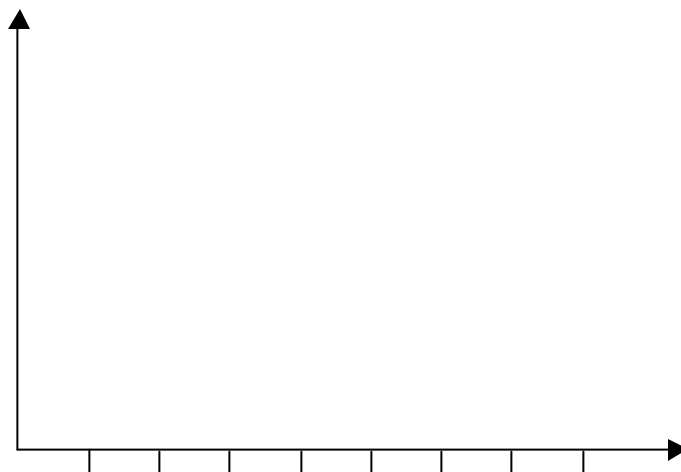
$$\text{seq}(X, X, 1, 8, 1) \text{\textcircled{R}} \text{L1}$$

2. Now we need to generate values of the response variable in list L2. The true regression line of this population is $y = 6$ with an error structure that is distributed as $N(0, 3)$. The following command puts 8 observations from this population in list L2.

$$6 + \text{randNorm}(0, 1, 8) \text{\textcircled{R}} \text{L2}$$

3. Fill in the table below with your observations. Round to two decimal places. Create a scatter plot by hand on the axes provided.

L1 (x)	L2 (y)



4. Calculate the least squares regression line for the above data using your calculator and record it below. Round the coefficients of the line to *three* decimal places.

5. We will use the letter b to denote the slope of the least squares regression line calculated from the sample data. What is the value of b for your least squares regression line?

6. Now, we want to test the null hypothesis that $b = 0$ using our value of b . We will let the calculator perform the linear regression t -test. Use your calculator to do this now.

7. Record the p -value from your test below. Round to three decimal places.

8. Suppose we test our hypotheses with a significance level of $\alpha = 0.10$. (Note that this is a higher significance level than would normally be used in practice. We are using an α this large so that your class will see some rejections of the null hypothesis. An $\alpha = 0.01$ would not generate many rejections. Why?) Compare your p -value to the significance level and make a decision about the null hypothesis.

Now, note that the sample data were generated by assuming the value of the null hypothesis slope was our population slope. Thus, in this case we *know* the population slope b equals 0. If you rejected the null hypothesis, then you *know* that you have made a Type I error! If you failed to reject the null hypothesis, then you made a correct decision.

9. The calculator program **ROBUSTT** automates the process you just went through. In the program you set the significance level α and the sample size n . Right now set $\alpha = 0.10$ and $n = 8$. Use the data recording worksheet to record the results of running the program 25 times. Be sure to record your results in the appropriate column.

10. After simulating 25 t -tests using the calculator program, how many times did you reject the null hypothesis? Is this more or less than the number you would expect?

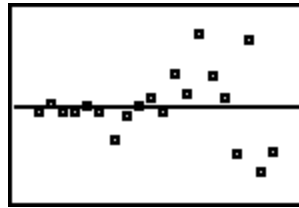
11. Report your results to your teacher so they can be combined with other student results. Ideally, the rate at which your class rejects the null hypothesis should equal $\alpha = 0.10$. Why?

The preceding situation had all conditions for the t -test satisfied. Now let's see a case when one of the conditions is not satisfied.

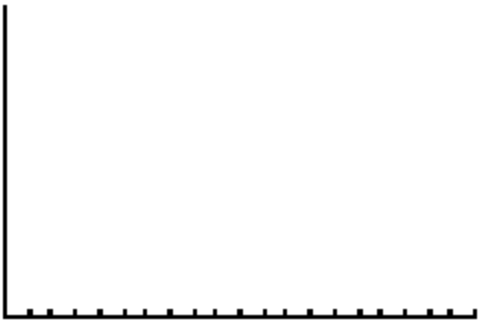
PART II
Condition violated – *The standard deviation of the errors is NOT constant*

In this section, we will generate data in the same manner as in the previous section, *but* with *one* exception: The standard deviation of the errors will *not* be constant for all values of x . To illustrate this condition, consider the following situation.

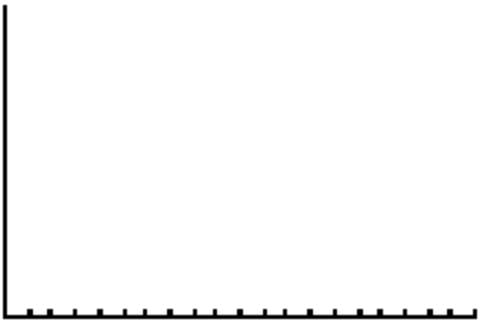
- Below is a *residual plot* for a sample of 20 observations from an unknown population. Note that as x increases, the values of the residuals get larger in absolute value. This could indicate a violation of the **constant standard deviation of errors** condition.



12. On the first set of axes below sketch a scatter plot that is *positively* associated and could have the residual plot drawn above. On the second set of axes sketch a scatter plot that is *negatively* associated and could have the residual plot drawn above.



Positively associated



Negatively associated

Now we will consider a population that has condition that was just illustrated. Again, consider two variables that have no relationship, and therefore a regression line with a population slope \mathbf{b} of zero. Knowing this, let us consider what would happen if we performed a test of significance on the slope. How often would we incorrectly reject the null hypothesis when, in fact, it is true? Let us test the null hypothesis

$$H_o : \mathbf{b} = 0 \quad \text{vs.} \quad H_A : \mathbf{b} \neq 0$$

As in Part I, you will generate sample data under the assumption that $\mathbf{b} = 0$. You will generate 8 observations from a population that has a regression line $y = 6$. **But** now the error structure will not have a constant standard deviation. The error structure from Part I (normally distributed with $\mu = 0$ and $\sigma = 1$) will be multiplied by the value of x . Why will this violate the **constant standard deviation of errors** condition?

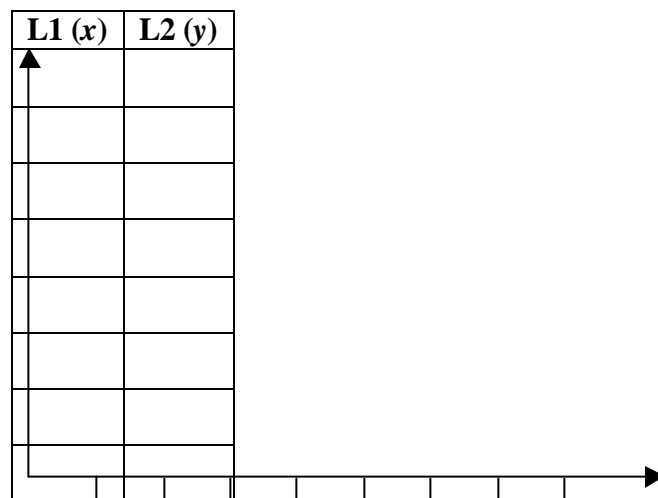
13. The values of the explanatory variable will again be integers from 1 to 8. If they already are in list L1, move to the next question. If not, use your calculator to create values of an explanatory variable in list L1. The command seq can be found under 2nd ® STAT ® OPS. (Or, of course, you can type these numbers into list L1.)

$$\text{seq}(X, X, 1, 8, 1) \text{ ® L1}$$

14. Now we need to generate values of the response variable in list L2. The true regression line of this population is $y = 6$ with an error structure that is distributed as $N(0, 1)$ times the value of the explanatory variable x . The following command puts 8 observations from this population in list L2.

$$6 + L1 * \text{randNorm}(0, 1, 8) \text{ ® L2}$$

15. Fill in the table below with your observations. Round to two decimal places. Create a scatter plot by hand on the axes provided.



16. Calculate the least squares regression line for the above data using your calculator and record it below. Round the coefficients of the line to *three* decimal places.

17. Again, use the letter b to denote the slope of the least squares regression line calculated from the sample data. What is the value of b for your least squares regression line?

18. Now, we want to test the null hypothesis that $b = 0$ using our value of b . We will let the calculator perform the linear regression t -test. Use your calculator to do this now.

19. Record the p -value from your test below. Round to three decimal places.

20. Suppose we use a significance level of $\alpha = 0.10$. Compare your p -value to the significance level and make a decision about the null hypothesis.

As in Part I, note the sample data were generated by assuming the value of the null hypothesis slope was our population slope. Thus, in this case we *know* the population slope b equals 0. If you rejected the null hypothesis, then you *know* that you have made a Type I error! If you failed to reject the null hypothesis, then you made a correct decision.

21. The calculator program **ROBUSTT** automates the process you just went through. In the program you set the significance level α and the sample size n . As in Part I, let $\alpha = 0.10$ and $n = 8$. Use the data recording worksheet to record the results of running the program 25 times. Be sure to record your results in the appropriate column.

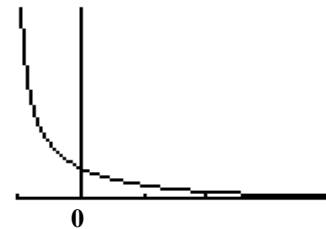
22. After simulating 25 t -tests using the calculator program, how many times did you reject the null hypothesis? Is this more or less than the number you would expect?

23. Report your results to your teacher so they can be combined with other student results. *If* the t -test is **robust**, the probability of incorrectly rejecting the null hypothesis – making a Type I error – should be close to the stated significance level α . What do the results of your class show?

The next section will look at a scenario where the **constant standard deviation of errors** condition is satisfied but the **normality of errors** condition is violated.

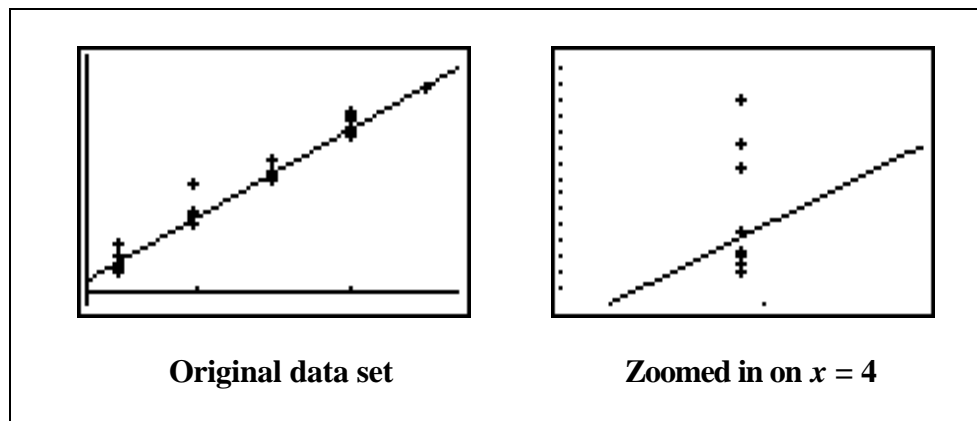
PART III
Condition violated – For fixed x , the errors are NOT normally distributed

In this part, we will simulate data using the same framework as in the previous two parts. The twist here is that for a fixed value of x , the standard deviation of the errors will **not** be normally distributed. The errors will follow the distribution pictured by the density curve at right, which has a mean of 0.

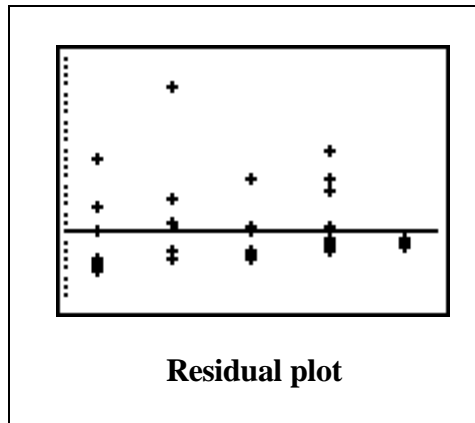


What happens when the errors come from a strongly skewed right distribution? Consider the following scenario as an illustration of this situation:

- We have a population where we have 5 values of x (1, 2, 3, 4, 5). For each value of x , we sample 8 observations of y . The errors follow the skewed right distribution pictured above. A scatter plot is provided at left below.
- To illustrate how a skewed error distribution affects the data, consider what happens at an individual x value. The graphs below show this condition. Note how the points under the line are close to the line and the points above the line are further away. This mimics the skewness of the error distribution.



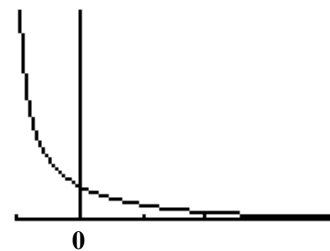
- To see what happens at each of the five x values, the residual plot for this data set is presented below. Note how the residuals above the line tend to be larger in absolute value than those below the line.



Now that you have seen how the violation of the **normality of errors** condition affects data visually, again consider two variables that have no relationship, and therefore a regression line with a population slope b of zero. Knowing this, let us consider what would happen if we performed a test of significance on the slope. How often would we incorrectly reject the null hypothesis when, in fact, it is true? One more time, test the null hypothesis

$$H_0 : b = 0 \quad \text{vs.} \quad H_A : b \neq 0$$

As in Parts I and II, you will generate sample data under the assumption that $b = 0$. You will generate 8 observations from a population that has a regression line $y = 6$. **But** now the error structure will **not** be normally distributed. It will follow the skewed right distribution whose density curve is pictured at right. This skewed right distribution has a mean of 0.



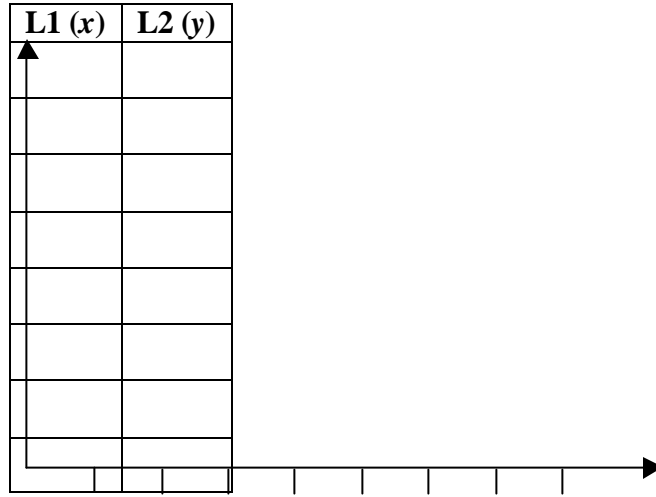
- The values of the explanatory variable will again be integers from 1 to 8. If they already are in list L1, move to the next question. If not, use your calculator to create values of an explanatory variable in list L1. The command SEQ can be found under 2nd Ⓜ STAT Ⓜ OPS. (Or, of course, you can type these numbers into list L1.)

$$\text{seq}(X, X, 1, 8, 1) \text{ Ⓜ L1}$$

25. Now we need to generate the values of the response variable in list L2. The true regression line of this population is $y = 6$ with an error structure that is skewed right with a mean of 0. The following command puts 8 observations from this population in list L2. (For an explanation of why, see below.¹)

$$6 + ((\text{randNorm}(0, 1, 8))^2 - 1) \text{ @ } L2$$

26. Fill in the table below with your observations. Round to two decimal places. Create a scatter plot by hand on the axes provided.



27. Calculate the least squares regression line for the above data using your calculator and record it below. Round the coefficients of the line to *three* decimal places.

28. Again, use the letter b to denote the slope of the least squares regression line calculated from the sample data. What is the value of b for your least squares regression line?

¹ The skewed distribution of errors we are using is a *chi-square distribution with one degree of freedom* whose density curve is shifted one unit to the left. This distribution has a mean of 0. Squaring the normal distribution with mean 0 and standard deviation 1 ($N(0, 1)$) yields a chi-square distribution with one degree of freedom, which has a mean of 1. We subtract 1 so that the mean of the chi-square distribution will be zero.

29. Now, we want to test the null hypothesis that $b = 0$ using our value of b . We will let the calculator perform the linear regression t -test. Use your calculator to do this now.
30. Record the p -value from your test below. Round to three decimal places.
31. Suppose we use a significance level of $\alpha = 0.10$. Compare your p -value to the significance level and make a decision about the null hypothesis.

Once again, note that the sample data were generated by assuming the value of the null hypothesis slope was the population slope. Thus, in this case we *know* the population slope b equals 0. If you rejected the null hypothesis, then you *know* that you have made a Type I error! If you failed to reject the null hypothesis, then you made a correct decision.

32. The calculator program **ROBUSTT** automates the process you just went through. As in Parts I and II, let $\alpha = 0.10$ and $n = 8$. Use the data recording worksheet to record the results of running the program 25 times.
33. After simulating 25 t -tests using the calculator program, how many times did you reject the null hypothesis? Is this more or less than the number you would expect?
34. Report your results to your teacher so they can be combined with other student results.

OK, so what was the point of doing all that work?

If a test is *robust*, then violating conditions of the test should not cause the probability of a Type I error to differ from the stated significance level α . The more robust a test is, the less effect violations of the conditions will have on the probability of a Type I error.

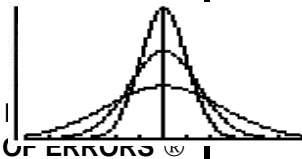
Your class has simulated several hundred t -tests when you *knew* that the null hypothesis was true. Thus, you can compare the proportion of times your class rejected the null hypothesis for each of the three situations with the stated significance level α . Fill in the table below with the *combined results of your class*. What do they indicate about the robustness of the t -test for the population slope when $\alpha = 0.10$ and $n = 8$?

DISTRIBUTION OF ERRORS ®	NORMAL		NORMAL WITH INCREASING ST. DEV.		SKEWED RIGHT DISTRIBUTION	
	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho	Reject Ho	Fail to Reject Ho
DECISION						
TOTALS						
% REJECTED Ho						

***t*-test for the slope of the least squares regression line**

Alpha =	
Sample Size =	

	CONDITIONS SATISFIED		ST. DEV. OF y INCREASES AS x INCREASES		ERRORS ARE STRONGLY SKEWED	
	NORMAL		NORMAL WITH INCREASING ST. DEV.		CHI-SQUARE (DF = 1)	
	Reject H_0	Fail to Reject H_0	Reject H_0	Fail to Reject H_0	Reject H_0	Fail to Reject H_0
TRIAL						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						
TOTALS						



% REJECTED H_0			
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