Using Simulations to Explore Random Walks

Name: _____ Class: _____ Date: _____

PURPOSE: This exploration allows you to use simulations to explore the outcomes of random walks.

MATERIALS NEEDED: TI-83 calculator and/or spreadsheet (able to read Excel files)
WALK2 program for TI-83
WALK spreadsheet file
This worksheet

GOAL: Upon completion of this exploration, you should be able to

1. explain how the number of moves allowed in a random walk is related to the final distance away from the starting point

2. explain how the number of moves allowed in a random walk is related to the probability of returning to the starting point

PROCEDURES:

1. Imagine that you are at the origin, (0, 0), of a coordinate system. You decide to take a walk on this system where each move is made according to the following conditions:

    a. You only move one unit at a time.
    b. You only move horizontally or vertically from where you are.
    c. For each move in the walk, one of the four possible directions is selected at **random**.

2. At the end of the allotted number of moves, you calculate your diagonal distance from your starting point, the origin.

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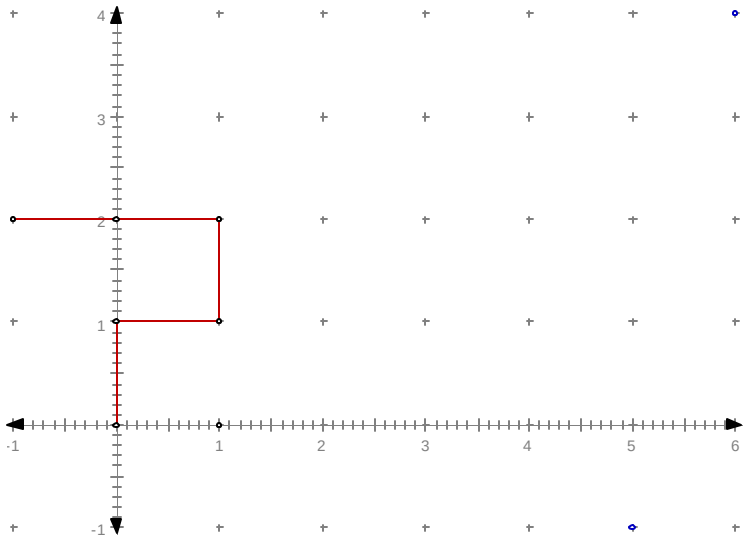
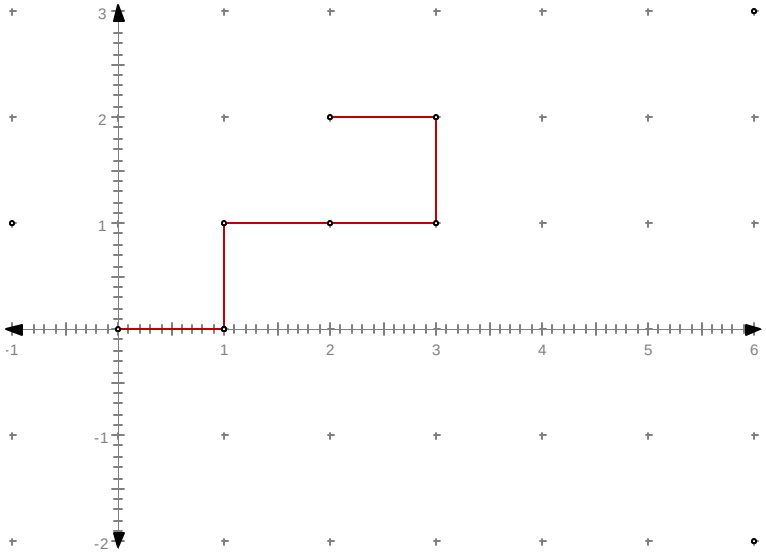
3. Examples:

Here is a picture of a random walk with five moves. The walk ends at (-1, 2). The distance from the starting point, (0, 0) is approximately 2.234 ($\sqrt{1^2+2^2}$).





Here is a picture of another random walk with six moves. The walk ends at (2, 2). The distance from the starting point, (0, 0) is approximately 2.828 ($\sqrt{2^2+2^2}$).

4. You can use either the WALK2 program for the TI-83 calculator or the WALK spreadsheet (Excel) file.

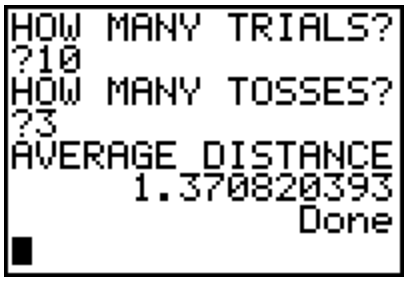
5. If you use the calculator program, you will need to keep a record of the outcomes. If you use the spreadsheet file, it keeps a record of the outcomes as well as a record of all of the points visited in the random walk.

6. The calculator program requires two inputs from you:

    a. the number of trials to run

    b. the number of tosses (random moves) in the walk

7. Here is a sample screen shot of the **calculator** program's output for the average (mean) distance away from the origin for 10 trials of three moves each:



Note that the average distance away from the origin for the 10 calculator trials is 1.37 units.

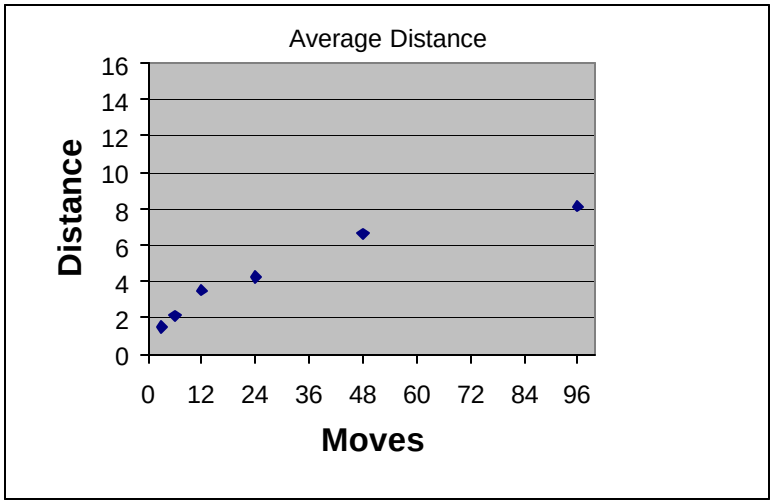
8. Here is a sample screen shot of the **spreadsheet** file's output (Summary tab) for the average (mean) distance away from the origin for 10 trials using 3 moves, 6 moves, 12 moves, etc.:

An Exploration of A Random Walk

In this exploration, you will take a RANDOM WALK. You start at the origin, (0, 0). On each move, you can move one unit in the horizontal direction from where you are or one unit in the vertical direction from where you are.

Press F9 key to generate new values

Number of Moves	Average Distance
3	1.494
6	2.139
12	3.529
24	4.234
48	6.603
96	8.130



9. Note that the average distance away from the origin for this set of 10 spreadsheet trials of 3 moves each is 1.49 units.

10. Your task is to explore either (or both) simulation environment(s) and determine:

    a. What are the variables in the simulation? What types of variables are they?

    b. What is the relationship, if any, between the variables? Gather enough data points so that you can use a calculator (or software program) to express the relationship, if it exists, using a regression curve and equation.

    c. Print your graph and equation.

11. Prepare a short presentation that discusses the variation in random walks of different lengths (number of moves).

References/Credits

- Walk2 program for the TI-83 calculator courtesy of Profs. George Reese and Jim Dildine, University of Illinois at Urbana-Champaign, Office for Mathematics, Science and Technology Education (www.mste.uiuc.edu/statmod)

EXTENSION: This section requires that you use the spreadsheet file WALK.

1. The **Summary tab** of the spreadsheet shows the average (mean) distances from the starting point for random walks of various lengths (number of moves).

2. The **Moves tab** of the spreadsheet give the coordinates of each move for each of the 10 random walks at each of the various random walk lengths (number of moves). Also displayed are the diagonal distance from the starting point for each trial and the average (mean) distance from the starting point for each set of 10 trials.

3. You can use the coordinates, if you wish (or your teacher instructs), to actually plot any or all random walks of interest.

4. As you have already noticed, random walks are not included for lengths of 4 moves, 5 moves, 7 moves, etc. You (or your teacher) may wish to add to the spreadsheet the capability to track random walks of other lengths of interest.

5. Use the coordinate data and distance data in the **Moves tab** to investigate the outcomes of random walks of various lengths (numbers of moves).

6. For what number ($n \geq 3$) of random moves is it most likely that you return to the starting point, (0, 0) at some time during the walk? With what probability?

7. For what number ($n \geq 3$) of random moves is it most likely that you end the walk at the starting point, (0, 0) ? With what probability?

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