

# Inference: Early and often

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## Rationale for this talk

- Inference = most difficult intro stats topic
- Logic of inference takes several days
- Learning to do inference takes months
- Why make students wait half the course?
- Dick Schaeffer's "plausible reasoning"
- George Cobb's "Ptolemaic curriculum"

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## Session objectives

- Make a compelling case for inference concepts being taught early and often
- Share classroom-tested, simulation-based inference activities
- Examine the meaning of: "could this result have happened just by chance?"

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## Scenario #1: Distracted driving

*Source: 2007 AP Statistics exam, Q5*

Researchers want to determine whether drivers are significantly more distracted while driving when using a cell phone than when talking to a passenger in the car. In a study involving 48 people, 24 people were randomly assigned to drive in a driving simulator while using a cell phone. The remaining 24 were assigned to drive in the driving simulator while talking to a passenger in the simulator. Part of the driving simulation for both groups involved asking drivers to exit the freeway at a particular exit. *In the study, 7 of the 24 cell phone users missed the exit, while 2 of the 24 talking to a passenger missed the exit.*

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## Let's look at the data

Here is a comparative bar graph of the data.



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## Distracted driving: conclusions?

- Can researchers conclude that cell phone use *caused* greater driver distraction?
- Well-designed experiments can help establish causation
- This was a well-designed experiment.
- There's a pretty large difference in the response variable between the groups.
- So...

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## Is there another explanation?

- Another possible explanation: differences weren't caused by difference in treatments
- Assume no difference in effects of these treatments on drivers
- Then each driver would make or miss the exit regardless of treatment assigned
- Any difference between the groups would be due to the random assignment

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## Blame it on random assignment?

- Assume no difference in treatment effects
- Randomly assign individuals to groups
- Examine what happens by chance
- Compare with observed results
- Could the result from the experiment be simply due to chance?

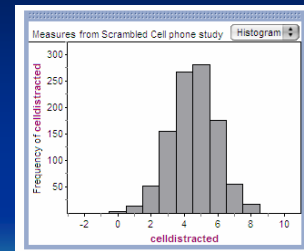
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## Rerandomizing simulation

- Want 9 “successes” and 39 “failures”
- Remove 10, J, Q, K of clubs
- A-9 of clubs = success  
everything else = failure
- “Shuffle up and deal” 24 cards for the cell phone group
- Record whether you get 7 or more “yes”es
- Pool results (via transparency)

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## Shuffling is fun, but...



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## Reprise: cell phones & driving

- Research question?  
Are drivers more distracted when using a cell phone than when talking to a passenger?
- Idea: suppose there’s no “treatment effect”
- Differences due to random assignment?
- “Re-randomize” many times
- What would you conclude?

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## Setting #2: Sleep deprivation

*Source:* Rossman et. al NSF Project

Researchers have established that sleep deprivation has a harmful effect on visual learning. But do these effects linger for several days, or can a person “make up” for sleep deprivation by getting a full night’s sleep on subsequent nights? A recent study investigated this question by randomly assigning 21 subjects to one of two groups: one group was deprived of sleep on the night following training and pre-testing with a visual discrimination task, and the other group was permitted unrestricted sleep on that first night. Both groups were then allowed as much sleep as they wanted on the following two nights. All subjects were then re-tested on the third day.

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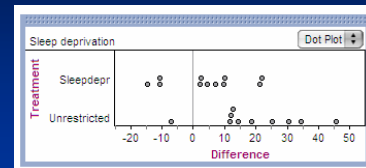
## Sleep deprivation data

Subjects' performance on the test was recorded as the minimum time (in milliseconds) between stimuli appearing on a computer screen for which they could accurately report what they had seen on the screen.

- Sleep deprivation ( $n = 11$ ): -14.7, -10.7, -10.7, 2.2, 2.4, 4.5, 7.2, 9.6, 10.0, 21.3, 21.8
- Unrestricted sleep ( $n = 10$ ): -7.0, 11.6, 12.1, 12.6, 14.5, 18.6, 25.2, 30.5, 34.5, 45.6

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## Did sleep deprivation cause difference in performance?



Or is there another possible explanation?

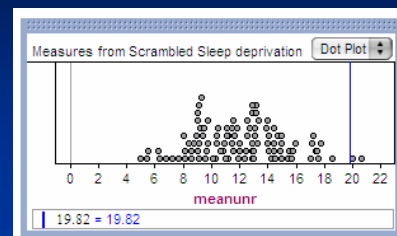
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## Rerandomizing simulation

- Place 21 cards (subjects) in a bag
- If no difference in treatment effects, then values same as in original study
- How large a difference in group means with different random assignments?
- Mix your cards and draw 10 to represent the unrestricted group.
- Compare your mean to 19.82. Report.

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## Physical simulation is fun, but...



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## Reprise: sleep deprivation

- Research question?  
Do the effects of sleep deprivation on visual learning last for several days?
- Idea: suppose there's no "treatment effect"
- Differences due to random assignment?
- "Re-randomize" many times
- What would you conclude?

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## Benefits of simulation-based inference

- Accessible to students from Day 1
- No distributions required
- "Could this happen just by chance?"
- Can be revisited often
- Allows study of means and proportions
- Connects to logic of inference

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## Inference: early and often

- George Cobb's 3 "R"s: Randomize, Repeat, Reject?
- Not restricted to randomized experiments
- Can spend time emphasizing:
  - Could this happen just by chance
  - Random sampling for generalizability
  - Random assignment to establish causation
- Don't wait! Try something now.

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## Epilogue

- I hope you found all this simulating rather stimulating...

**Please share your experiences**  
**E-mail: [dstarnes@lawrenceville.org](mailto:dstarnes@lawrenceville.org)**

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