Precalculus Recursively-Defined Equations Lab

A simple model that can be used for one species with two age classes is the following,

$$C_n = A_{n-1}\exp(r - aC_{n-1} - bA_{n-1})$$
$$A_n = kC_{n-1}$$

where C_n represents the population (density) for the adolescence in the nth-generation and A_n represents the population (density) for the adults in the nth-generation. There are four parameters: r, a, b and k. One can think of r as the amount of resources available to the species. The parameters, a and b, are used to quantify how much resources the adolescence and adults use, respectively. Finally, the parameter k is the survival rate of the adolescence to adulthood.

For the lab, consider the simplified system where the number of the parameters is reduced,

$$C_n = A_{n-1}\exp(r - aC_{n-1} - A_{n-1})$$
$$A_n = C_{n-1}$$
.

Note that this system only has two parameters, r and a. Use this simplified system to answer the following questions.

1. Using your calculator investigate how this system behaves for various values of r and a. To start you may want to let $r = 1$ and try values for a between 0 and 2. Next try increasing r and again try some values for a. Each time try to determine the behavior of the system by picking various "initial conditions" and determining where these initial conditions go. In other words, pick different values for r and a and different starting points for C_0 and A_0 and iterate the system and find out what happens to the C's and the A's. Do they go to an equilibrium or to something else? For each pair of values for r and a you will want to pick at least a few, if not several values for C_0 and A_0. You should always start with $C_0, A_0 > 0$, why?

   It is important to document what you do, so make a chart of what values of r and a you used, and the different initial conditions that you used, AND what happened to each initial condition.

2. Now try r <0 and various values for a. What happens to the system now? Is anything different? Can you explain the differences mathematically or biologically?

3. Next try a <0 for the values for the same values in part 1. Does the system behave the same way? If not, can you make a conjecture as to why not? What do you think a <0 represents biologically?

4. For extra credit change the model to reflect some interesting biological phenomena. Make some conjectures as to how you think this will change the behavior of the system. Finally, repeat steps 1, 2 and 3 to see how accurate your conjectures were.