

# 10 Games

Teaching Contemporary Mathematics Conference  
The North Carolina School of Science and Mathematics  
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Bridging games:

1. The Bridges of Bangalore
2. Hex

Number theory games:

3. Sprouts
4. Nim

Variation on an old favorite:

5. 3-player tic-tac-toe

Deductive strategy games:

6. Trailblazer
7. Battleship

Card games:

8. Strategic War
9. 11-card Bluff
10. Eleusis

A downloadable copy of this handout is available on-line at the web site for the Teaching Contemporary Mathematics Conference, a link to which can be found at <http://courses.ncssm.edu/math/>

# 1. The Bridges of Bangalore

for 2 players

## *Rules:*

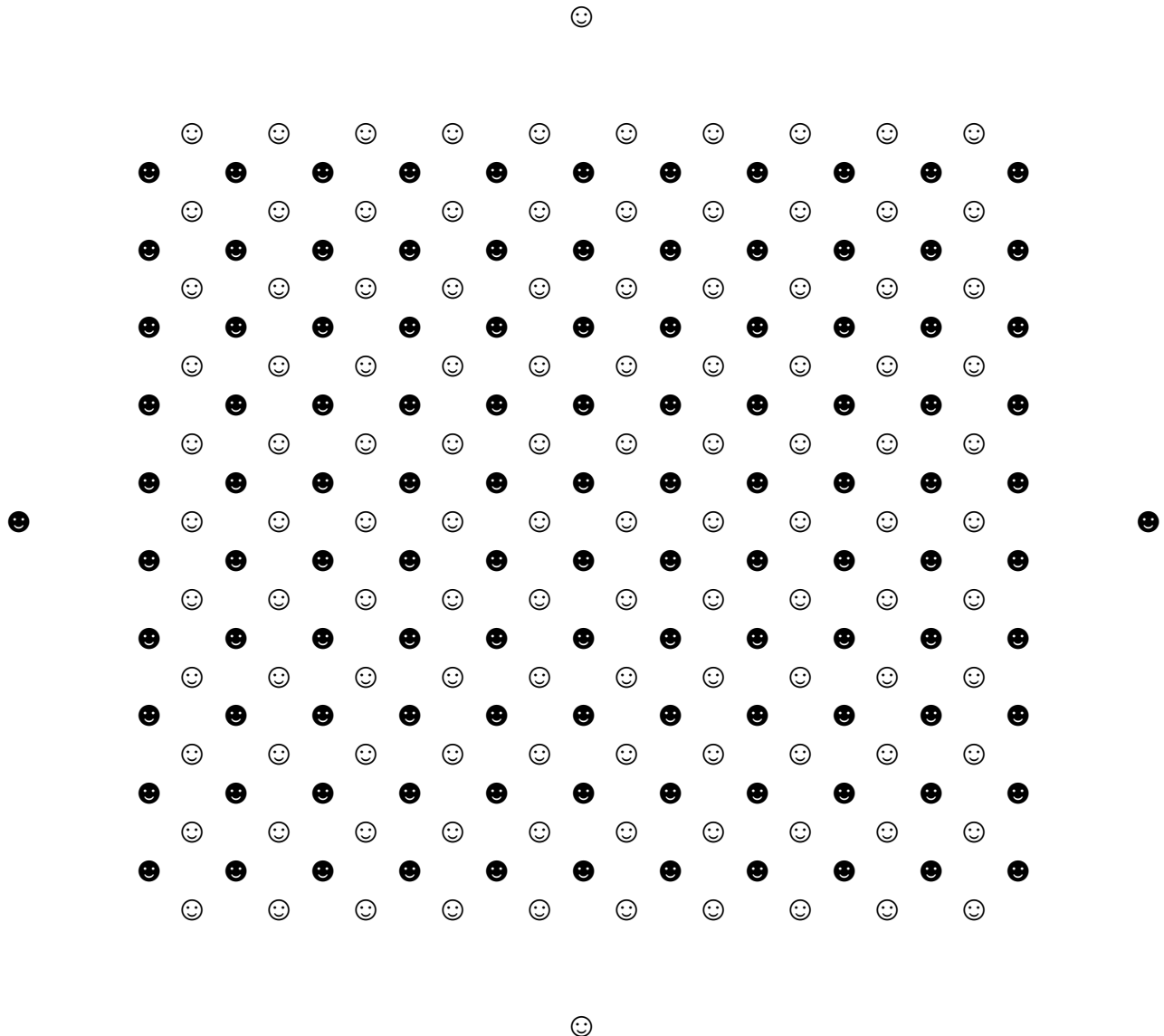
Players alternate turns. One player connects adjacent ☺'s with one color pen, the other player connects adjacent ☹'s with another color pen. (The different colors are not required, but they make the game much easier to play.) A bridge need not necessarily connect to any bridges already played.

All bridges must be either horizontal or vertical, and none may cross an opponent's bridge.

The player who connects his two sides with an unbroken string of bridges (even if it bends and twists) wins.

## *Tip:*

Don't give up even if it seems hopeless. Remarkable comebacks occur surprisingly often.



## 2. Hex

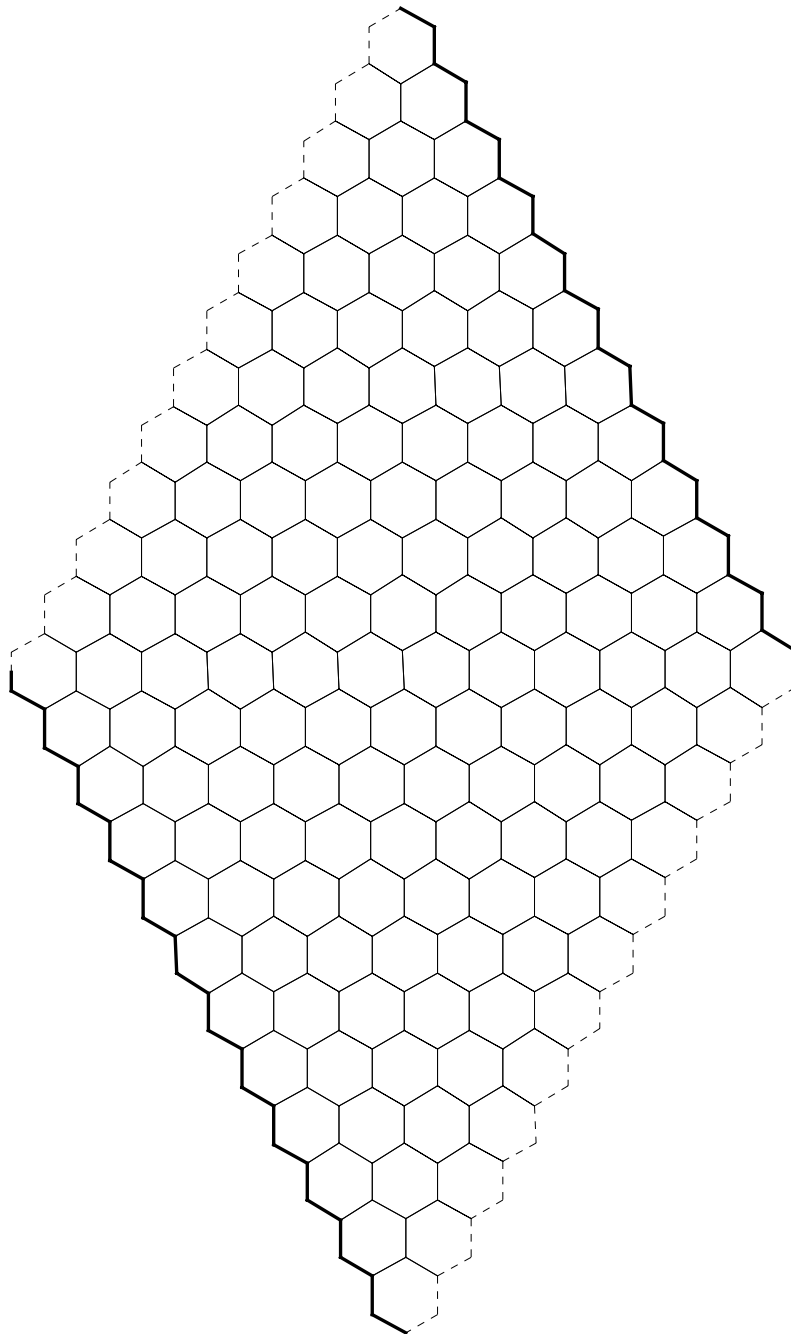
for 2 players

### *Rules:*

One player's goal is to connect one pair of opposite sides (marked with dark segments on the board below) and the other player's goal is to connect the other pair of opposite sides (marked with dashed segments). Sides are chosen before beginning play.

Players alternate turns. Each player on his turn puts his or her mark in any empty hexagon. (Different colored pens make the game easier to play, although X's and O's may be used.)

The first player to connect his or her two sides of the game diamond with an unbroken string of hexagons (even if it bends and twists) wins.



### 3. Sprouts

for 2 players

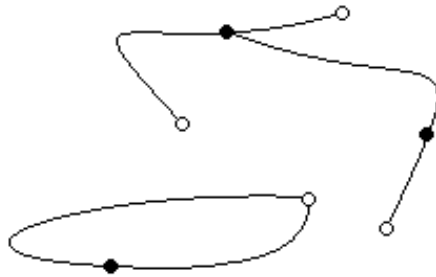
*Rules:*

The Game of Sprouts (invented by John Conway who also invented the Game of Life and Surreal Numbers) begins with  $n$  distinct dots, known as vertices or nodes (5 is a good number) placed at random on a page. A move consists of drawing an arc (called an edge) from one dot either to itself or to another dot, and then placing a new dot somewhere on the arc. Only two rules must be observed:

- (1) No edge may cross itself, a previously drawn edge, or pass through a node.
- (2) No node may have more than three edges emanating from it.

Players take turns drawing edges. The winner is the last player able to draw an edge.

A game in progress is shown below. This game began with the four dots that are drawn as open circles. The darkened dots were drawn later. Note that one node has been “used up”.



## 4. Nim

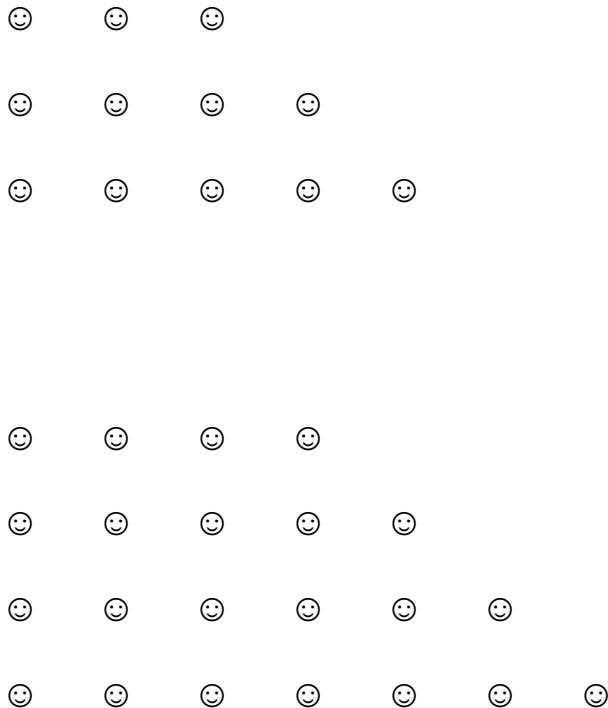
for 2 players

### *Rules:*

The game of Nim begins with several rows of dots. A row of 3, a row of 4 and a row of 5 is typical. Each player on his turn eliminates as many dots as he wishes from a single horizontal row. The winner is the player who eliminates the last dot. (Traditionally, Nim is played with rows of coins or tokens that are removed in turn.)

### *Variation:*

In the *misère* version of Nim, the winner is the player who forces his opponent to eliminate the last dot.

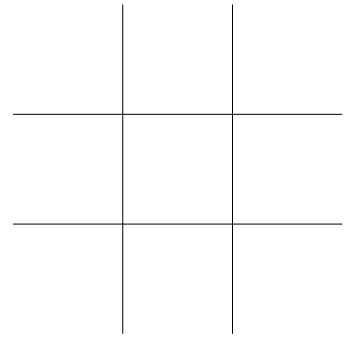
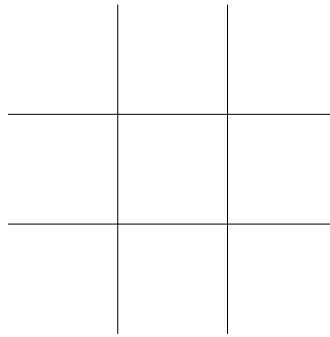
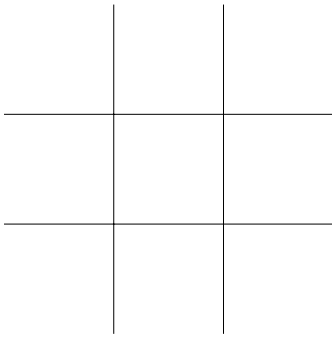


## 5. Three-player tic-tac-toe

for 3 players

### *Rules:*

In this variation on tic-tac-toe, three players (identified before play as X, O, and “cat”) consecutively place X’s or O’s in a tic-tac-toe board. The three players play in turn and the mark alternates between X and O. (Thus, a single player does *not* always place the same mark.) When the board is completed, the X player wins if there are three X’s in a row, the O player wins if there are three O’s in a row, and “cat” wins if no row contains three X’s or three O’s. (A “row”, as in tic-tac-toe, may be horizontal, vertical, or diagonal.)



## 6. Trailblazer

for 2 players

### Rules:

In this game, each player has his own sheet of paper that he keeps hidden from his opponent. The game begins with each player secretly hiding a trail in a 6×6 grid. The trail must have the following characteristics:

- The first square in the trail must be an integer from 1 to 25. It may be placed anywhere in the grid.
- Beginning with that first square, a trail of increasing consecutive integers are placed in further squares, each adjacent to its predecessor. (Throughout these rules, “adjacent” means adjacent horizontally or vertically.)
- Some squares may be left empty.
- No two empty squares may be adjacent to one another, no empty square may be adjacent to the beginning or the end of the trail, and the beginning of the trail may not be adjacent to the end of the trail.

Once both players have hidden their trails, play begins. Players alternate turns. On a player’s turn, he identifies a single square by its coordinates and his opponent reveals what number is in the square, or reveals that it is blank.

When a player believes he knows his opponent’s entire grid, he declares it at the beginning of his turn, before asking for any new square. The two grids are compared and if his grid is entirely correct he wins. However, if he was the first player to go, then his opponent gets one more turn and the game may end in a tie.

### Example:

	1	2	3	4	5	6
A	18	17	46	45	40	39
B	19	16	15	44	41	38
C	20	21	14	43	42	37
D	23	22	29	30		36
E	24		28	31	34	35
F	25	26	27	32	33	

### My Trail:

	1	2	3	4	5	6
A						
B						
C						
D						
E						
F						

### My Opponent’s Trail:

	1	2	3	4	5	6
A						
B						
C						
D						
E						
F						

## 7. Battleship

for 2 players

### *Rules:*

This game has much of the flavor of Trailblazer but is more violent. Each player has his own piece of paper that he keeps hidden from his opponent.

Each player begins with two 10×10 grids, one representing his portion of the ocean and the other representing his opponent's portion of the ocean. On his own grid, he hides a battleship, a cruiser, and two destroyers. The locations of these are not known to his opponent. The battleship is four consecutive squares long, the cruiser is three, and each of the destroyers is two. A ship may sit horizontally, vertically, or diagonally on the game board. Different ships may be next to one another but may not occupy the same square.

After both players have secretly placed their ships, play begins. Players alternate turns. On a player's turn, he drops as many bombs into his opponent's ocean as there are remaining ship squares to be hit. For example, on his first turn, a player drops  $4+3+2+2=11$  bombs. If these bombs hit, say, the battleship twice and a destroyer once, then on his next turn, he will drop 8 bombs. A bomb is dropped by identifying the coordinates of the square into which the bomb is targeted.

A player announces the locations of *all* of his bomb drops, and then his opponent tells him what ships were hit, if any, and how many times. The opponent does *not* reveal which bombs made the hits.

A ship is sunk when all of its squares have been bombed. Turns alternate until one player wins by sinking all of his opponent's ships. (If he was the first player to go, then his opponent gets one more turn and the game may end in a tie.)

### *Tips:*

When dropping or receiving a round of bombs, mark the grid with numbers to indicate the turn number, rather than with indistinguishable X's. Later, if you know that you hit your opponent's battleship once during round 1 and twice during round 5, then you have a pattern to look for: a 1 and two 5's.

If you think you know the location of one of your opponent's ships, you may not want to sink it right away. Saving it for later gives you more bombs during each of the interim turns.

**My Ocean, My Ships:**

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>A</b>										
<b>B</b>										
<b>C</b>										
<b>D</b>										
<b>E</b>										
<b>F</b>										
<b>G</b>										
<b>H</b>										
<b>I</b>										
<b>J</b>										

**My Opponent's Ocean, My Opponent's Ships:**

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>	<b>10</b>
<b>A</b>										
<b>B</b>										
<b>C</b>										
<b>D</b>										
<b>E</b>										
<b>F</b>										
<b>G</b>										
<b>H</b>										
<b>I</b>										
<b>J</b>										

**Mark here the turn numbers in which you hit your opponent's ships:**

Battleship				
Cruiser				
Destroyer 1				
Destroyer 2				

## **8. Strategic War**

for 2 players

### *Rules:*

This game is played with a standard deck of playing cards. Players begin the game by separating the four suits. Each player chooses a suit and receives all the cards of that suit. (It is easier to play if the chosen suits are of the same color.) A player's cards are her armament. A third suit is chosen, and those cards are the spoils of war. The fourth suit is not used.

In all three of the suits used, aces are low and are worth one point only. 2's through 10's are worth their face values, jack are worth 11 points, queens are worth 12 points, and kings are worth 13 points.

The spoils of war cards are shuffled and placed face down between the players. The top card is then turned up and each player chooses a card from his hand with which to try to win the face up card. The players put their cards face down on the table in front of them. A player may change his mind about what card to play until the moment when both players have a card face down at the same time. At that point, they turn their cards face up and the player with the higher card wins the face-up card from the spoils of war pile. She places that card in front of her face down, and both players place their used armament cards face down in front of them in a discard pile. (For a game that requires some strategy but less memory, leave all played cards face up.)

If there is a tie, neither player wins the spoils of war card.

After all the cards have been played, the player with the higher total of the spoils of war cards wins.

## 9. 11-card Bluff

for 2 players

### *Rules:*

From a standard deck of cards, all the cards of one suit are used except for the queen and king. Any set of cards numbered from 1 through 11 will work as well.

The cards are shuffled and five cards are dealt to each player. The eleventh card is placed face down between them.

Players alternate taking turns. On a player's turn, he may either ask his opponent whether she holds a certain card, *or* he may guess the value of the face-down card. He may not do both in one turn, however. When asked by her opponent whether she holds a certain card, a player must answer truthfully.

If a player correctly identifies the hidden card, he wins. If he incorrectly guesses it, he loses.

### *Tip:*

There is some psychology involved in playing this game well. Asking for a card that you yourself hold cannot result in a gain of information; but by doing so, you *may* trick your opponent into incorrectly guessing the value of the hidden card on her next turn.

## **10. Eleusis**

### *A Game of Induction for 4-8 Players*

This game stands out like a sore thumb among the other nine. It is complex and they are simple. It involves scoring multiple rounds while they stand alone as short games. It requires at least 4 players while most of them are two-player games. It requires two decks of cards while most of them require merely paper and pencil. I include it quite simply because it is an exceptionally intriguing game that has great appeal to mathematicians. Many find it absolutely addictive. Details about the game can be found in the book *Penrose Tiles to Trapdoor Ciphers*, by Martin Gardner.

#### ***Rules:***

Two decks of cards are shuffled together. From 4 to 8 players may play. A large playing space is needed.

#### **The Oracle's Task**

One player is selected (either by general agreement or by highest card drawn) to be the Oracle. The Oracle comes up with a rule by which a sequence of cards may lead to a legal play. Rules should be neither too simple nor too hard. (The scoring system favors the Oracle when someone else is able to induce the rule, but not too soon.) Examples of good rules are:

- If the last legally played card was odd, play a black card. Otherwise play a red one.
- If the last legally played card was black, play a card of equal or higher value. If the last legally played card was red, play a card of equal or lower value. (The values of the jack, queen, king, and ace are respectively 11, 12, 13, and 1.)
- Spades and hearts must be followed by either diamonds or clubs, and vice-versa. (Note that alternating black and red cards would probably be too simple a rule.)

The secret rule must be written down in unambiguous language and tucked away for future reference. For advanced players, a rule may be created that involves the entire sequence of cards currently on the table. (For example, "If there are an odd number of hearts on the table, then play a card greater than 7. Otherwise play a card less than or equal to 7.") But such rules are not advisable for new players, and under *no* circumstances may the rule involve anything other than the sequence of legally played cards. (For example, the rule may *not* be "Play a red card if the Oracle has her fingers to her lips.") If she wishes, the Oracle may give a hint about the rule. *e.g.*, "The rule involves color but not suit."

The Oracle shuffles the two decks of cards and deals 11 cards to each player. She then plays a card, called the starter card, at the left edge of the playing space. Beginning with the player to the Oracle's left, the Oracle counts from 1 to the value of the starter card. That determines who plays first. Play then proceeds to each player's left, always skipping the Oracle, who plays no cards.

## The Players' Task

On a player's turn, he holds up a card to the Oracle. If the card is a legal card (following the starter card and the other legal cards already on the table), the Oracle says Yes, and it goes to the right of the last legally played card in the sequence. If it is not a legal card, the Oracle says No, and the card is placed below (not physically beneath) the last legally played card for future reference ("this card was not legal"). A player proposing an illegal card receives two cards from the Oracle, who holds all the unused cards. If two or more illegal cards are proposed by consecutive players, they are lined up below the last legally played card.

Scoring favors getting rid of cards in your hand. If a player thinks he knows the rule and wants to play several cards at once, he may do so, proposing a string of 2, 3, or 4 cards to the Oracle. The Oracle then either accepts them all in the proposed order or else rejects them all without identifying the offending card or cards. If they are legal, they are played in order in the horizontal line of legal cards. If they are illegal, they are played below as before, but overlapping so that players will see that they were played simultaneously. In the case that they are not legal, the player who attempted to play them receives twice as many cards from the Oracle as he attempted to play.

If a player runs out of cards then the round ends.

If a player believes he has no legal play in his hand, he may say so. In that case, he shows his cards to everyone and the Oracle determines whether he is correct or not. If he is incorrect and there is a legal play, then the Oracle makes a legal play from his hand and penalizes him by dealing him 5 more cards. If he is correct in declaring "no play", then he is dealt a fresh hand with four fewer cards than he had before. If that leaves him with no cards then he is out of cards and the round ends.

## The Prophet's Task

There is a second way to score many points, other than getting rid of your cards. A player who thinks he knows the secret rule may declare himself a Prophet, but only if the following conditions are met:

1. He has just played (correctly or incorrectly) and the next player has not yet played.
2. There is not already a Prophet.
3. At least two other players besides the Oracle and himself are still in the round.
4. He has not been a Prophet before in this round.

If a player becomes a Prophet, then he ceases playing cards and his turn gets skipped just like the Oracle's. A marker is placed on the last card played to indicate at what point in time the Prophet declared himself. (This is relevant in scoring.) He then takes over the Oracle's job of deeming legal or illegal the plays by the other players. The Oracle supervises to be sure that Prophet is making correct judgments.

If a Prophet makes an incorrect call, then the Oracle declares him to be a false prophet, removes his marker from the playing space, and deals him 5 cards. In that case, however, no other player is penalized, even if he proposed an illegal play. The false prophet returns to play on his turn and cannot again be a prophet during this round.

## The End of a Round

A round of play ends when a player runs out of cards either by making a legal play with his last card or cards, or else by calling “no play” correctly when he holds four or fewer cards.

However, after 30 cards have been played (either correctly or incorrectly) and there is no Prophet in the game, players are *expelled* from the round when they make a wrong play or incorrectly declare “no play”. An expelled player is given the appropriate number of penalty cards (which will be used only for scoring) and plays no more while the other players continue.

If there is a Prophet, expulsions are delayed until at least 20 cards have been played (again, correctly or incorrectly) after the Prophet’s marker.

If all players have been expelled (excluding a Prophet, if there is one) then the round is over.

In rare circumstances, both decks of cards may run out while the game is still going on. A third deck may be introduced, or expelled players may count their cards (for future scoring) and hand them to the Oracle for recycling.

## Scoring

The greatest number of cards held by anyone (including the Prophet) is called the “high count”. Each player (including the Prophet) subtracts the number of cards in his hand from the high count. The difference is his score. If he has no cards, he gets a bonus of 4 points.

The Prophet, if there is one at the end of a round, also gets a bonus. It is the number of legal cards played after he became Prophet plus twice the number of illegal cards played since he became a Prophet.

The Oracle’s score equals the highest score of any player, including bonuses. There is one exception: If there is a Prophet, count the number of cards (legal and illegal) that precede the Prophet’s marker and double that number. If the result is smaller than the highest score, the dealer’s score is that smaller number.

Ideally, rounds are played until each player has been the Oracle once. If the game must end before each player has had a chance to be the Oracle, then anyone who did not get to be the Oracle receives 10 extra points. Each player’s score for all rounds is totaled and the winner is the player with the highest score.

**Example of a game in progress.** The rule here is one of the three given as examples under the section “The Oracle’s Task”

A♠	3♣	K♠	4♦	4♥	A♦	9♥	10♣	Q♠	8♣
2♠		9♠	5♣	Q♥	J♦		10♦	6♠	
		2♥		9♥					
		7♠							