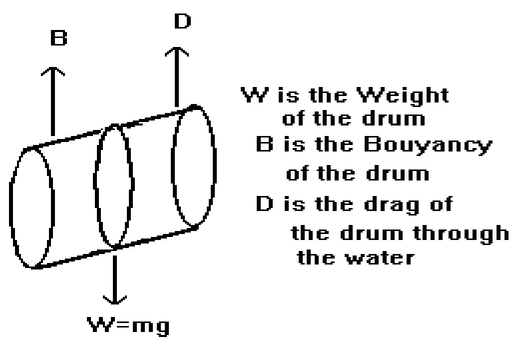
Nuclear Waste Disposal

For many years, the United States "solved" its nuclear waste disposal problem by filling 55 gallon drums with the waste and dropping them off in the ocean at a depth of 300 feet. When questioned about the practice, the Atomic Energy Commission (now the Nuclear Regulatory Commission or NRC) argued that the drums would never leak from prolonged exposure to the sea. While satisfied about the deterioration of the drums, a number of engineers questioned whether the drums would withstand the impact when they land on the bottom of the ocean.

Tests were done, and the engineers found that if the barrel landed with a velocity greater than 40 feet per second, the drums were susceptible to rupture. How can we determine the velocity of a barrel after it has fallen through salt water to a depth of 300 feet?

What do we know about this situation? We always know that $F = ma$. Further, we know that $a = \frac{dv}{dt}$. So we always have the differential equation $\frac{dv}{dt} = \frac{F}{m}$ at our disposal. We will use this equation often. In each physical situation, the forces are different, so we need to determine specific expressions for *F* and *m*. What are the forces acting on the drum as it falls?



The force causing the drum to descend is due to gravity, and is measured by the *weight* of the drum. The weight of a 55 gallon drum filled with nuclear waste is estimated to be 527 pounds. Two forces act to retard the descent. The first is the *bouyancy* of the drum. Bouyancy is the force of the water acting on the drum. Its magnitude is the weight of the water displaced by the drum. A 55 gallon drum filled with sea water weighs approximately 470 pounds. The second force is the *drag* force of the water acting on the drum. The drag force depends upon the velocity of the object moving through the water, the faster it moves, the greater the resistance. By using towing experiments, the relationship between the drag force and velocity of the drum was determined to be $D = .08v$. The orientation of the drum as it fell did not appear to effect this relationship. Naturally, other objects moving through other mediums will have a different functional representation for drag.

Putting this all together gives us the following pieces of information:

$$\frac{dv}{dt} = \frac{F}{m} \text{ (always).}$$

In this specific case, we know that

$$F = W - B - D = (527 - 470 - 0.08\,v),$$

and that

$$W = mg \quad \text{so} \quad m = \frac{527}{32.2}.$$

This gives the differential equation $\frac{dv}{dt} = \frac{32.2}{527} \cdot (57 - 0.08v)$.

Unfortunately, at this point in the course, we cannot solve this equation exactly. However, we can use Euler's Method to solve it approximately.

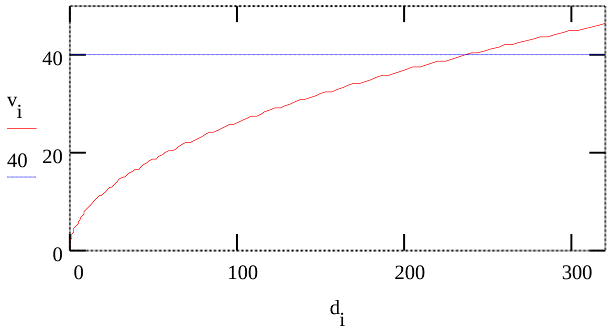
We cannot solve $\frac{dv}{dt} = f(v)$ (in this case $f(v) = \frac{32.2}{527} \cdot (57 - 0.08\,v)$ but we won't write it all out to save space). So we solve the approximate equation $\frac{\Delta v}{\Delta t} = f(v)$ instead. We know from this approximation that $\Delta v = f(v) \cdot \Delta t$. Remember, Δv is the difference in successive v-values, so $\Delta v = v_n - v_{n-1}$. This generates the equation $v_n - v_{n-1} = f(v_i) \cdot \Delta t$. This is finally rewritten as Euler's formula $v_n = v_{n-1} + f(v_{n-1}) \cdot \Delta t$.

This iterative equation generates velocities as a function of *time*. We want to know the velocity as a function of *distance*, specifically, what is *v* when *d* is 300! How do we get from velocity to distance. What is the relationship between velocity and distance?

Since velocity is the derivative of disance with respect to time, we know that $\frac{d}{dt} d = v$. This is another differential equation. We solve the approximate equation $\frac{\Delta d}{\Delta t} = v$ as before.

$$\Delta d = v \cdot \Delta t \quad \rightarrow \quad d_n - d_{n-1} = v_{n-1} \cdot \Delta t \quad \rightarrow \quad d_n = d_{n-1} + v_{n-1} \cdot \Delta t.$$

This iterative equation uses the values generated previously for velocity to generate depths. We now graph *v* against *d*, and see how fast the drum is moving at 300 feet.



$t_{133} = 13.3$ $t_{134} = 13.4$

$d_{133} = 299.291$ $d_{134} = 303.777$

$v_{133} = 44.857$ $v_{134} = 45.184$

Velocity as a function of distance, with 40 ft/sec illustrated

As can be seen from the graph above, the velocity at 300 feet is approximately 45 feet per second; too fast for the Atomic Energy Commission, which put an end to dumping of nuclear waste in the ocean.

References:

Bartkovich, Kevin, John Goebel, Julie Graves, and Daniel Teague, *Contemporary Calculus through Applications*, Everyday Learning Corporation, Chicago, 1995.

Braun, Martin. *Differential Equations and Their Applications*, 3rd Ed., Springer-Verlag New York, Inc., New York, 1983