

Investigating the Spread of the H1N1 Flu

Human history is crowded with devastating epidemics.

In the 14th century, there were an estimated 25 million deaths in a population of 100 million Europeans attributed to an epidemic of bubonic plague. In 1520, the Aztecs suffered an epidemic of smallpox that resulted in the death of half their population of 3.5 million. When measles first came to the Fiji Islands in 1875 as a result of a trip to Australia by the King of Fiji and his son, it caused the death of 40,000 people in a population of 150,000. In the three-year period from 1918 to 1921, there were an estimated 25 million cases of typhus in the Soviet Union and about 1 in 10 victims died from the disease. In a world-wide epidemic of influenza in 1919, more than 20 million persons perished from the illness and subsequent attacks of pneumonia.

Olinick, An Introduction to Mathematical Models in the Social Sciences, 1978, page 349

The number of deaths from SARS, AIDS and H1N1 (“Swine”) flu pales in comparison to these historical epidemics, but the threat of new outbreaks is a part of our daily lives. In this investigation, we will explore mathematical models describing the spread of an infectious disease such as the H1N1 Flu that has spread around the world in recent weeks.

The model for the spread of infectious diseases is known as a compartment model, since it is useful to think of people moving from one compartment to another. Assume there is a population of N individuals (N is constant over time) through which the flu is moving. The people who have the disease are called *Infecteds*. The people who do not yet have the disease but may catch it if they interact with an infected person are called *Susceptibles*. Some of the people may have already had the disease and have recovered from it; they are called *Recovereds*. For some diseases, the *Recovereds* develop immunity to the disease, while for others they return to the susceptible group and can again come down with the disease. This model is illustrated in Figure 1.

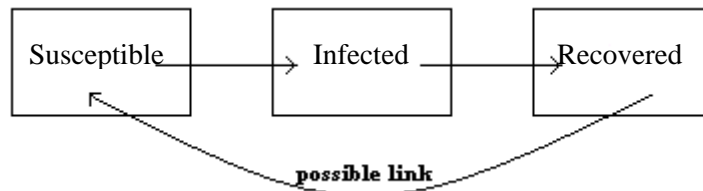


Figure 1: A Compartment Model

Compartment models can be modeled effectively by recursive equations with the form $NewValue = OldValue + Gain - Loss$. If $Y(t)$ represents the number of people in a particular group at time t , then $Gain - Loss$ represents the net change in the size of the group. That is, $\frac{dY}{dt} = Gain - Loss$.

In order to begin our investigation, we need to consider how to model the change in the number of *Susceptibles*, *Infecteds* and *Recoveredds* over time. Where should we begin? Well, we do know the following:

- there is no “gain” for *Susceptibles*;
- there is no “loss” for *Recoveredds* (we are assuming immunity once individuals have recovered);
- the “gain” for *Infecteds* is the “loss” for *Susceptibles*; and
- the loss for *Infecteds* is the gain for *Recoveredds*.

Our differential equations thus need to incorporate a rate of recovery, β . A proportion of *Infecteds* will recover; this means that they can no longer spread the disease and are immune to catching it again.

Unfortunately, for some diseases, “recovery” may mean death due to the disease. For non-fatal diseases, if

infected individuals have the disease for k days, the recovery rate can be estimated as $\beta = \frac{1}{k}$.

If we let α represent the rate of the flu’s transmission, how would we define our differential equations representing the change in the number of *Susceptibles*, *Infecteds* and *Recoveredds* over time?

$$\frac{dS}{dt} = \underline{\hspace{2cm}}$$

$$\frac{dI}{dt} = \underline{\hspace{2cm}}$$

$$\frac{dR}{dt} = \underline{\hspace{2cm}}$$

Now that we have our differential equations, we can begin our investigation...

Numerical Investigation with Euler's Method

1. Use Euler's method to generate values and graphs of S, I, and R. Start with the following initial values and constants: $N = 50,000$, $S_0 = 49,990$, $I_0 = 10$, $\alpha = 0.000005$ and $\beta = \frac{1}{7}$. Use $\Delta t = 1$.
2. According to your Euler's method approximations, when is the flu spreading most rapidly?
3. How does the spread of the flu change if $N = 20,000$? if $N = 10,000$? if $N = 1,000$? Assume $I_0 = 10$ in all cases. For each of these population sizes, estimate:
 - the total number of individuals who eventually become ill;
 - the maximum number of persons who are ill at one time;
 - the time at which the flu is spreading most rapidly.
4. At the present time, patients are treated with antibiotics, but no one knows if this treatment is effective. Returning to $N = 50,000$, suppose a treatment is found that cuts the recovery time in half. What is the effect on the value of β in the differential equations? How does this affect the total number who become ill, the maximum number ill at one time, and the time at which the flu is spreading most rapidly?
5. For the most part, we can't really affect the value of β very much. Recovery time is often little affected by our actions. However, the transmission rate can be affected in a number of ways. Returning to $\beta = \frac{1}{7}$, we will now investigate the consequences of changing the transmission rate.
 - a. Suppose the flu is more difficult to catch, i.e. that the transmission rate is lower. Use $\alpha = 0.0000025$. How does this affect the total number who become ill? What is the effect on the maximum number ill at one time?
 - b. Suppose the flu is easy to catch as measles, which would imply that $\alpha = 0.00001$. How does this affect the total number of people who become ill and the maximum number ill at one time?
 - c. Many people in areas where an infectious disease is widespread try to reduce the spread of the disease by wearing masks. The Canadian Broadcast Corporation (CBC) did a report of the ability of masks to reduce the number of microscopic particles breathed in. They found that a standard dust mask reduced the number of airborne particles by 13 percent, and an N-95 mask (like those used by the Army in Iraq) reduced these particles by 98 percent. Could wearing either of these masks significantly alter the spread of the flu?
 - d. How low must α be before fewer than half of the population actually gets ill?

Analytical Investigation

1. According to the differential equation for $\frac{dI}{dt}$, under what conditions will $I(t)$ be increasing? decreasing? What does this mean about the spread of the flu? Using this result, explain whether quarantine will be effective against the flu.

2. Use the differential equations for $\frac{dI}{dt}$ and $\frac{dS}{dt}$ to find $\frac{dI}{dS}$. Solve this differential equation for $I(S)$.

Based on $\frac{dI}{dS}$ and $\frac{d^2I}{dS^2}$, determine when the number of infected will begin to decrease. Compare this to your solution from # 1.

3. Use the function $I(S)$ to determine the maximum number of individuals ill at one time. What proportion of the population will be sick simultaneously? Are these results consistent with your results using Euler's method?
4. When the epidemic ends, how many people had not ever contracted the flu? (*Hint*: What would have to be true about $I(S)$ for the epidemic to be considered over?)